Improved Decision Making Under Uncertainty: Incorporating a Monte Carlo Simulation into a Discounted Cash Flow Valuation for Equities

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Abstract

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Discounted cash flow valuation (DCF) models are a common approach to valuing equities and traditionally they aim to provide the most likely outcome as a single point estimate. However, this approach ignores the uncertainty related to the drivers of value for a security. In order to gain a more accurate representation of the intrinsic value of a security, the uncertainty related to the inputs of the model should be included in the valuation.

Monte Carlo simulation is a method used in modeling systems that are affected by randomness. Through random sampling, multiple scenarios can be generated and the nature of the randomness can be assessed. Monte Carlo simulation is often used in solving problems for which there is no analytical solution. The idea behind the method is to iterate a process thousands of times with random input variables to attain a probability distribution of all possible outcomes instead of a single point estimate. In the context of valuation, this method can be used to sample future cash flows, which can then be discounted, resulting in a distribution of possible values for the security.

This study aims introduce a method for incorporating a Monte Carlo simulation aspect into a DCF valuation model and to then examine the benefits of such an approach. The theoretical framework presents the basic theories of discounted cash flow valuation and an introduction to the Monte Carlo simulation method. In the empirical section of this study I describe the model used and present the valuations I performed on several companies.

The findings of this study were that by putting the valuation in context with the uncertainty related to the asset, the investment decision could be improved. By viewing the upside the valuation model implies in relation to the amount of uncertainty related to the models inputs, provides a new aspect to the decision making. The investor can view the results not only through the expected return but also through the level of uncertainty.

Keywords
Equity valuation, discounted cash flow valuation, Monte Carlo simulation

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1 Introduction

Discounted cash flow valuation (DCF) models are a common approach to valuing equities and traditionally they aim to provide the most likely outcome as a single point estimate. However, this approach ignores the uncertainty related to the drivers of value for a security. In order to gain a more accurate representation of the intrinsic value of a security, the uncertainty related to the inputs of the model should be included in the valuation.

Monte Carlo simulation is a method used in modeling systems that are affected by randomness. Through random sampling, multiple scenarios can be generated and the nature of the randomness can be assessed. Monte Carlo simulation is often used in solving problems for which there is no analytical solution. The idea behind the method is to iterate a process thousands of times with random input variables to attain a probability distribution of all possible outcomes instead of a single point estimate. (Brandimarte 2014, 3; Benninga 2014, 605.) In the context of valuation, this method can be used to sample future cash flows, which can then be discounted, resulting in a distribution of possible values for the security.

This study introduces a method for incorporating a Monte Carlo simulation aspect into a DCF valuation model and then examines the benefits of such an approach. The theoretical framework presents the basic theories of discounted cash flow valuation and an introduction to the Monte Carlo simulation method. In the empirical section of this study I describe the model used and present the valuations I performed on several companies.

1.1 Objectives and scope of this study

The objective of this study was to construct a discounted cash flow model for equities and incorporate a Monte Carlo simulation method to gain more insight into the distribution of the expected intrinsic value of the company being valued rather than a single, most likely point estimate. The research problem concerned whether it is possible to enhance investment decisions through this process. I constructed the model and performed valuations on several companies, after which I compared the results. Since the objective focused more on methods and models, a detailed analysis of company or sector specifics was excluded from the scope of the study.

1.2 The structure of this study

This study is divided into four sections. In the first section the objective and the research question are presented. The second section forms the theoretical framework for the study and it consists of a presentation of the basic concepts of equity valuation and discounted cash flow valuation. The major components of a DCF valuation and a more in-depth introduction to the Monte Carlo simulation method and how it can be applied to a DCF valuation are also presented. The third section describes how a Monte Carlo simulation DCF model based on the theories from the theoretical framework was constructed and used to run simulations on several companies, with the objective of determining whether the research question could be answered by the results. The conclusions that can be drawn from these results are presented in the fourth section.

2 Theoretical Framework

In this section of the study I describe the underlying theories for valuing equities and for Monte Carlo simulation. This is the basis for the model constructed in the third section.

2.1 Equity valuation

In general, the process for valuing a company can be broken down into five steps. 1) First you perform an analysis of the industry, competitors, financial statements and other company disclosures to provide a solid basis for forecasts. After gaining a thorough understanding of the business, you then proceed to forecast the future performance of the company, which then will be used as inputs for the chosen valuation model. The third step involves selecting the valuation model and the aim is to find the one which most accurately captures the characteristics of the company being valued. The fourth step is converting these forecasts into valuations, and the final step is evaluating the results and reaching a conclusion. Through this process, the investor aims to gain a fairly accurate estimate of the value of the company and possibly discover mispricings in the market to generate abnormal returns or alpha. (Pinto, Henry, Robinson, Stowe & Rath 2010, 7-26.)

Valuing any security is always an estimation and a number of assumptions are often included. Assumptions are biased by our preconceptions of the security in question and the most challenging part in any valuation is overcoming these biases. (Damodaran 2012c, 3.)

2.1.1 Valuation models

Valuation models can be divided into absolute and relative valuation models. Relative models are based on the assumption that similar assets should sell for a similar price. (Damodaran 2003b, 12.) Absolute models aim to deliver an estimation of the intrinsic value of the asset and compare this estimate to the market price. The intrinsic value is said to be the “true” value of an asset, given a hypothetically complete understanding of its characteristics, and it can deviate from the market price of the asset. The most common type of absolute valuation model is the present value model or discounted cash flow model. The logic behind this type of model is the notion that the value of an asset must be related to the returns, or cash flows, the investor might expect by owning the asset. (Pinto & al. 2002, 85.)

2.1.2 Present value

It is commonly accepted that the fundamental value of a security is its expected future cash flows discounted to the present at a rate which accurately represents the risk involved. This concept is fairly straightforward when valuing assets such as bonds, for which the received coupon payments are predetermined and very few assumptions need to be made. However, the cash flows yielded by equities cannot be estimated with similar levels of confidence, and the amount of uncertainty related to the cash flows plays a major role in determining the value of the asset. Therefore, Monte Carlo simulation and scenario analysis are essential tools in including uncertainty into valuations. (Viebig, Poddig & Varmaz 2010, 8.)

2.1.3 Risk

Risk is defined as the variance of actual returns around the expected returns. When examining investments, the investor looks for the highest return for a given amount of variance, or risk. For investing in equities, risk can be divided into firm-specific and market risk, where firm-specific risks are unique to a certain company or a small group of companies and affect the expected cash flows. Examples of firm-specific risks are project risk, competitive risk and sector risk. Market risk, for example interest rate increase or weakening of the economy, affects the economy as a whole, and consequently, a majority of investments. By investing in a stock, the investor is exposed to both, firm-specific and market risk. However, the investor can reduce the firm-specific risk by diversifying or investing in a large portfolio of stocks. Therefore, once diversified, the investor is viewed to be exposed only to the market risk and when weighing new investment opportunities, the investor would only need to adjust for this risk. The long-term standard for accounting for market risk is the capital asset pricing model, or CAPM, and it assumes the additional risk an investment adds to the portfolio is captured through its beta. (Damodaran 2003b, 5-7.)

2.2 Discounted cash flow valuation

Discounted cash flow valuation is an absolute valuation method which aims to produce an intrinsic value estimation by discounting the expected future cash flows to the present at a rate which describes the risk associated with the asset. The concept can be expressed with the following equation (Pinto & al. 2010, 7-26.):

$$V_0 = \sum_{t=1}^{t=n} \frac{CF_t}{(1+r)^t}$$

Where:
V_0 = present value
n = life of the asset
CF_t = cashflow at period t
r = discount rate

The value of equity is largely based on the profitability of the company. Growth in sales and profit margins dictate the future cash flows, so discounted cash flow models have high sensitivity to these inputs. Also, the phase of growth the company is experiencing should be examined and the structure of the model and the assumptions used should be adjusted accordingly. Does the company have a high competitive advantage with profitability and cash flows expected to grow rapidly in the coming years? Or is it a maturing company with sales that are levelling off? To account for this, the discounted cash flow model can be structured into various different stage configurations.

The single stage or constant growth model assumes the cash flows grow at a constant rate until infinity. The main advantage is the simplicity of the model, but the premise might not be the most accurate for a majority of companies. (Damodaran 2002a, Chapter 13 1-4.)

Probably the most common variation for a discounted cash flow valuation model is the two-stage model. In the first stage of the model, the assumption is that the company has a period of time where it has an advantage over its competitors, thus the cash flows are expected to grow at an abnormally high rate compared to the growth rate of the economy. However, after the competitive advantage diminishes, the growth rate settles at a constant, sustainable rate until perpetuity. The second stage is also referred to as the terminal value. (Damodaran 2002a, Chapter 13 8-9.) The two-stage model can be expressed as:

$$V_0 = \sum_{t=1}^{t=n} \frac{CF_t}{(1+r)^t} + \frac{CF_{t+1}}{r-g} * \frac{1}{(1+r)^t}$$

Where:

V_0 = present value

n = life of the asset

CF_t = cashflow at period t

r = discount rate

g = perpetuity growth rate

(Pinto & al. 2010, 186.)

The three-stage model is very similar to the two-stage model except that the high-growth period is followed by a transitional phase, with a gradually declining growth rate down to the perpetuity rate. (Damodaran 2002a, Chapter 13 21-24.)

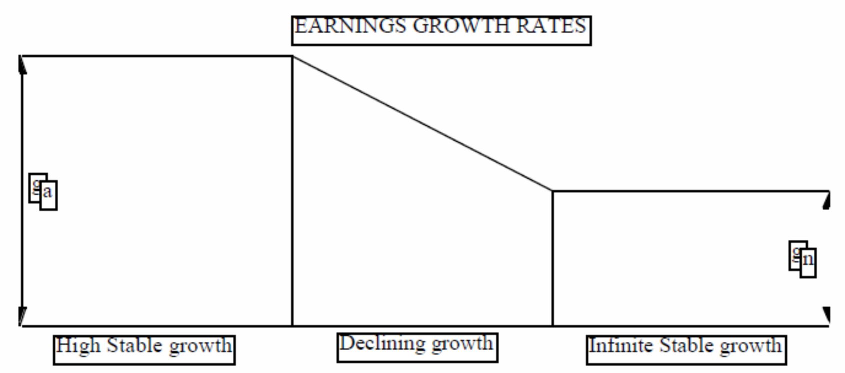

Figure 1. An illustration of the three-stage model (Damodaran 2002a, Chapter 13 24.)

These are the three most common approaches. However, the DCF model can be as complex and multi-staged as the valuator is willing to put time into it. Building a discounted cash flow model can be demanding, but once it is built, the ease of changing the inputs to more accurately match your views of the company and the possibility of tailoring it to a broad selection of different scenarios makes it a powerful tool.

2.2.1 Free cash flow

Free cash flow to the firm, or FCFF, is the amount of cash left from earnings, available for distribution amongst providers of capital, after operating expenses and investments in capital have been paid. Investments in capital can be divided into investments in working capital (e.g. inventory) and investments in fixed capital (property, plant and equipment).

Through these investments, companies aim to grow their asset base and revenues and ultimately cash flows in the future. After the investments, companies are free to distribute the excess cash amongst capital providers. The cash flows can be used to pay dividends, buy back shares, pay back debt or to fund acquisitions or mergers. The formula for calculating free cash flow to the firm is:

$$FCFF = EBIT * (1 - T) + D\&A - CAPEX - \Delta \mathrm{NWC}$$

Where:
FCFF = Free cash flow to the firm
EBIT = Earnings before interest
T = tax rate
D&A = Depreciation & amortization
CAPEX = Capital expenditures
∆NWC = Changes in net working capital

The basis for the calculation is often the earnings before interest and taxes, or EBIT, as it is widely considered to be the definition of operating profits. EBIT is revenues less the cost of goods sold, or COGS, and other operating expenses such as selling, general and administration and research and development. Tax expenses are included by multiplying the EBIT with (1 – tax rate). (Viebig, Poddig & Varmaz 2010, 27-28.)

Depreciation and amortization are income tax deductions to account for the aging of an asset throughout its lifetime. Since they are non-cash charges, meaning they are for accounting purposes and no cash is actually delivered, they should be added back after deducting tax expenses. (Pignataro 2013, 91.)

After these line items are added, the calculation of free cash flows takes the form:

$$FCFF = (Rev - COGS - OPEX) * (1 - t) + D\&A - CAPEX - \Delta \mathrm{NWC}$$

Where the added items are:
Rev = revenues
COGS = cost of goods sold
OPEX = other operating expenses (SG&A, R&D, D&A)

While FCFF is the cash left to all suppliers of capital, free cash flow to equity, or FCFE, is the cash left to holders of common equity. It is the cash left after all operating expenses, capital expenditures and payments to debt holders have been paid. FCFE can be calculated through FCFF by subtracting the after-tax interest expenses from it and adding net borrowings. Net borrowings is defined as the difference between debt issued and debt repaid. In essence, FCFE is the amount available to pay out as dividends. (Pinto & al. 2010, 163.)

$$FCFE = FCFF - I * (1 - T) + NB$$

Where:
FCFE = Free cash flow to equity
I = Interest expenses
T = tax rate
NB = Net Borrowings

2.2.2 The discount rate

The discount rate is a representation of the minimum amount of return the investor is demanding for the specified time period, considering the riskiness of the asset. Or, in other words, it is the return the investor can attain from investing in a different asset with the same level of riskiness. In order to justify an investment, then, the asset should yield more return on a specified risk level than is available elsewhere.

When valuing the entire firm, the appropriate discount rate is often considered to be the weighted average cost of capital or WACC. The WACC consists of the cost of equity and the cost of debt, each of which are weighted by the proportion of equity and debt. (Benninga 2014, 71-72.)

$$\mathrm{WACC} = \frac{E}{E + D} * R_E + \frac{D}{E + D} R_D \, (1 - T_C)$$

Where:
E = market value of firms equity
D = market value of firms debt
T_c = corporate tax rate
R_E = cost of equity
R_D = cost of debt

The standard model for the cost of equity is the Capital Asset Pricing Model, or CAPM, which measures the riskiness of an asset in relation to the market portfolio. The three inputs for the model are the risk-free rate, the market risk premium, which is the additional return over the risk-free rate the investor demands for investing in the market, and the assets beta, which is the covariance of the asset with the market, standardized by dividing it with the variance of the market. The beta measures the risk added by the investment. (Damodaran 2002a, Chapter 8 2.)

$$R_E = R_f + \beta (R_m - R_f)$$

Where:
R_E = required return
R_f = risk-free rate
β = beta
R_m = market expected return

The cost of equity is the expected return to the equity investor and, if valuing a company through FCFE, it is the appropriate discount rate since the cash flows have the payments to other capital providers already deducted. (Pinto & al. 2010, 163.)

2.2.3 Terminal value

As mentioned earlier, the value of a security is its cash flows from now until infinity, discounted at a rate that appropriately reflects the riskiness of the security. Since it is practically impossible to forecast cash flows until infinity, a common practice is to forecast the cash flows for a limited period, for which the cash flows grow at a pace that reflects the competitive advantage of the firm, after which the growth is assumed to revert to a constant rate until infinity. When choosing a growth rate, one must keep in mind that it cannot exceed the growth rate of the economy, since this would imply that at some point in the future the company would be worth more than the economy. An assumption often used is that in the long run, real interest rates and real economic growth rates converge, thus the risk-free rate can be used as an upper limit for the perpetuity growth rate. (Viebig, Poddig & Varmaz 2010, 45-46.)

The terminal value at the end of the competitive advantage period is:

$$\mathrm{TV}_t = \frac{FCFF_t * (1 + g)}{WACC - g}$$

Where:

TV_t = the terminal value at t

t = end of the competitive advantage period t

$FCFF_t$ = free cash flow at time t

g = the perpetuity growth rate

WACC = the weighted average cost of capital

Since this is the value at time t, this value must still be discounted to the present:

$$\mathrm{TV}_0 = \frac{TV_t}{(1 + WACC)^t}$$

Where:

TV_0 = present value of the terminal value

2.2.4 Equity value

The value of equity is defined as the enterprise value less net debt and current preferred and minority interest. The enterprise value is the present value of the expected free cash flows to the firm and the present value of the terminal value added. Net debt can be expressed as short-term debt and long-term debt less cash and marketable securities. The whole formula can be written as:

Equity Value =
+Present value of FCFF
+Present value of the terminal value
-Short-term debt
-Long-term debt
+Cash and marketable securities

To arrive at the equity value per share, this must be divided by the shares outstanding. (Benninga 2014, 123.)

2.3 Monte Carlo simulation

Monte Carlo simulation is a method used in modeling systems that are affected by randomness. Through random sampling, multiple scenarios can be generated and the nature of the randomness can be assessed. Monte Carlo simulation is often used in solving problems for which there is no analytical solution. Originally developed for uses in other areas such as physics, it has also been adopted by the finance industry. (Brandimarte 2014, 3.; Benninga 2014, 605) The idea behind the method is to iterate a process thousands of times with random input variables, from a distribution based on historical occurrences, to attain a probability distribution of all possible outcomes instead of a single point estimate. In the context of valuation, this method can be used to sample future cash flows, which can then be discounted to arrive at a distribution of possible values for the security (French & Gabrielli 2005, 7-8). This type of output can be beneficial when comparing two stocks that seem to be similarly valued relative to their price. However, if one of them has a significantly higher amount of uncertainty related to its inputs, a rational investor should choose the stock with the lower variance, since it offers the same return with more certainty.

The computational effort that a Monte Carlo simulation demands is high and in the past, computers able to handle the calculations were expensive, which meant access to them was scarce. A single point estimation approach was therefore the optimal choice. However, computing has come a long way since then and today the typical desktop computer has more than sufficient computing power for the application of a simulation. In addition to that, access to data has improved greatly through the internet.

2.3.1 Distribution types

The aim in choosing a distribution type is to find the one which most accurately describes the underlying data. In order to achieve this, the following four questions need to be answered: is the data discrete or continuous, is the data symmetrical or asymmetrical, are there upper or lower limits to the data, and what are the likelihoods of extreme values?

Discrete distribution has only a finite number of possible outcomes, while a continuous distribution has an infinite number of different outcomes within a range. (Damodaran n.d., 43-44.). The symmetry of a distribution concerns whether the distribution is skewed towards either side of the mean, which would indicate if a negative or positive outcome is more likely.

The most commonly used distribution is the normal distribution, which is a symmetrical continuous distribution. Its benefit lies in the ease of its implementation, since defining it requires only the mean and the standard deviation of the data. The probability of a certain value occurring can be obtained by examining how many standard deviations it is from the mean value. The normal distribution is best suited for cases in which the data is strongly concentrated around the mean, the data is symmetrical and extreme values are highly unlikely. (Damodaran n.d., 49-50.)

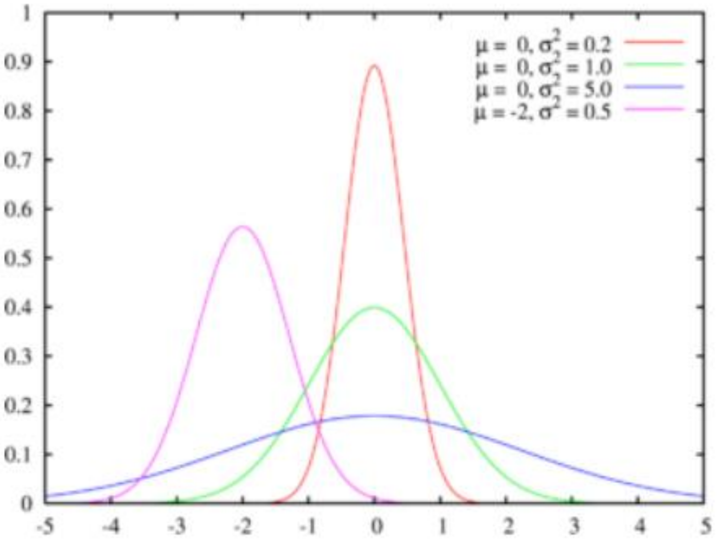

Figure 2. A Normal Distribution (Damodaran n.d., 50.)

If the data has upper or lower limits, the normal distribution might not be suitable since there is a possibility, although very small, for extreme values beyond these limits. For data with a lower limit of zero, the lognormal distribution can be used. Defining a lognormal distribution requires three parameters: shape, scale and shift.

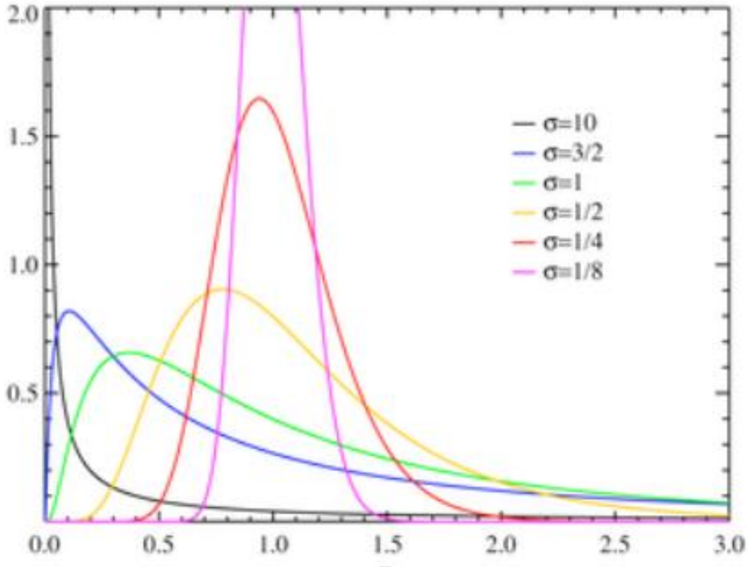

Figure 3. A Lognormal Distribution (Damodaran n.d., 53.)

For data resembling a normal distribution or a lognormal distribution, but with both lower and upper limits, the triangular distribution is suitable.

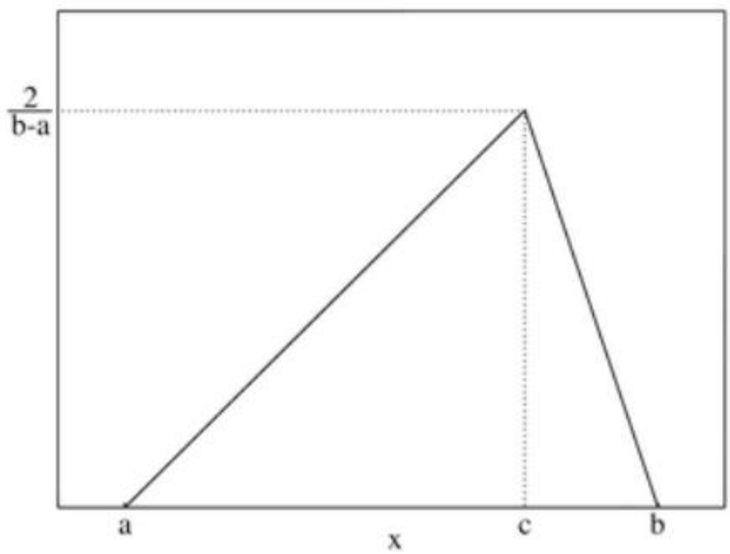

Figure 4. A Triangular Distribution (Damodaran n.d., 57.)

2.3.2 Monte Carlo simulation for DCF

In practice, and in the context of valuation, the simulation approach can be used to assign probability distributions to the inputs of our discounted cash flow model, instead of absolute, most likely scenario figures. As an example, take the growth rate of revenue for a company. Historically, the revenue has grown at a certain mean, but year-to-year it deviates around this mean at some average level. If the growth rate is assumed to be normally distributed, the mean and standard deviation form a distribution, which can be used as the input. Now the revenue for the next year can be forecasted by randomly selecting a growth rate from the distribution. This process can be continued for the following years, using the previously forecasted figure as the input for the next forecast. This forms a type of random walk and the chain can be iterated thousands of times. An iteration here is considered to be one full calculation where a single intrinsic value estimate is attained. Since the probability that a certain growth rate appears in the simulation is based on the historical distribution of the figure, it should mirror the uncertainty experienced in the real world. For revenue forecasts the process can be expressed as follows:

$$R_t = R_{t-1} * (1 + g_s)$$

Where:
R_t = the forecasted revenue at time t
t = year of the forecast
g_s = simulated revenue growth rate

For other line items in the cash flow calculations, instead of forecasting a rate at which the item will increase, the items need to be viewed in relation to the main driver, or revenue. For COGS, the simulated variable can be assigned as a percentage of revenue and how this percentage has deviated around the average for the past years. With these figures, a distribution can be formed. A random selection from this distribution, together with the simulated revenue estimate, can be used to arrive at a forecast for COGS:

$$COGS_t = R_t * COGS\%_s$$

Where:
$COGS_t$ = the forecasted cost of goods sold at time t
t = year of the forecast
$COGS\%_s$ = simulated cost of goods sold as a percentage of revenue

The same concept can be used for the remaining line items as well, and the calculation for the simulation can be expressed as follows:

$$FCFF_t = (R_{t-1} * (1 + g_s) - R_t * COGS\%_s - R_t * OPEX\%_s) * (1 - T_s) + R_t * D\&A\%_s - R_t * CAPEX\%_s - R_t * \Delta \mathrm{NWC}\%_s$$

Where:
$FCFF_t$ = free cash flow to the firm at time t

R_t = the forecasted revenue at time t

t = year of the forecast

g_s = simulated revenue growth rate

$COGS\%_s$ = simulated cost of goods sold as a percentage of revenue

$OPEX\%_s$ = simulated operating expenses as a percentage of revenue

T_s = simulated tax rate

$D\&A\%_s$ = simulated depreciation and amortization as a percentage of revenue

$CAPEX\%_s$ = simulated capital expenditures as a percentage of revenue

$\Delta NWC\%_s$ = simulated change in net working capital as a percentage of revenue

Once all the figures have been simulated, the free cash flow for the projected year can be calculated. This process can be repeated for the chosen number of years forward for the competitive advantage period and discount the simulated cash flow figures with the WACC. As DCF models are highly sensitive to changes in the discount rate, the discount rate should also be simulated and the formerly presented methods are also valid for the this. For every iteration, the present value of the terminal value needs to be calculated.

As an output, the simulation gives an array of possible values which can then be grouped into ranges. By counting the number of instances in every range and dividing it by the total number of simulations, the probability of the intrinsic value being in a certain range can be calculated.

3 Empirical Study

In this chapter I describe how I applied the theories from the previous chapters to build a discounted cash flow valuation model, with inputs defined as probability distributions and the valuation process involving a Monte Carlo simulation method, to ultimately arrive at a distribution for the intrinsic value of a company's stock. The second part of this chapter consists of valuations for a group of tech companies and an analysis of the results. I conducted valuations on Apple, Microsoft and IBM and examined whether any further insight into the characteristics of risk related to the asset could be attained.

3.1 Introduction to the model

The model is a discounted cash flow valuation model and aims to produce a probability distribution of the intrinsic value of a company being valued by incorporating a Monte Carlo simulation into the process.

The calculations were done for the whole firm using FCFF and discounting the cash flows with the WACC. The future cash flows were forecasted through stochastic variables based on historical occurrences. The data was imported from a Bloomberg terminal into an Excel spreadsheet and the model was constructed in a way that made it quick to retrieve the data and run simulations. The Bloomberg Excel add-in includes special formulas for importing data into the spreadsheet. By inputting the ticker for the company and the code for the specified field, the formula imports data straight into the cell. In the model, the formula for every data point was referenced to a single cell with the ticker for the stock being valued. This meant that by changing only the one cell, it was possible to import the exact same data for a different company. This made it quick to conduct valuations on many companies. Once the data was imported into the worksheet, I used VBA code to construct the simulation for the model. The simulation used 100,000 iterations.

The assumption for the inputs was that they are normally distributed. Although a better-fitting distribution could possibly be found, in my view and considering the aim of this study, the ease of implementation and higher degree of automation made the assumption of normal distribution a better choice.

For the free cash flow calculations, I gathered data from the past 11 years and used the historical data to arrive at estimates for the line items used 11 years forward. The growth rate of revenue was used as the main driver for the model. There were a few approaches that could have been used to forecast revenues, the simplest being to use the average growth rate of the past years as the input for the estimates. The method I used as a default was that for the first three years forward, I used the consensus estimates, which can be retrieved through the Bloomberg terminal, after which the growth rate would steadily decline to the long-term growth rate. However, I built in a manual override field for cases in which there is an out-of-consensus view, or when the consensus estimates are not available. These figures were then used as the mean when defining the distribution for the growth rate and the standard deviation was the historical standard deviation for the past 11 years, as this should be a sufficiently accurate measure of the uncertainty related to the line items. For the rest of the line items, the historical mean and standard deviation were used.

3.2 Case Study: comparing the valuations of Apple, Microsoft and IBM

In this section I present the valuations, the inputs used, the results and comparisons of the results for each of the companies. I chose Apple, Microsoft and IBM as the companies to be compared. I describe the main inputs used and the results for each company and present them in table and chart form.

3.2.1 Apple

Apple's revenue grew from 13.9 billion USD in 2005 to 233.7 billion USD in 2015. For the same time period, the FCFF margin grew steadily from 10.6% to 25.9%.

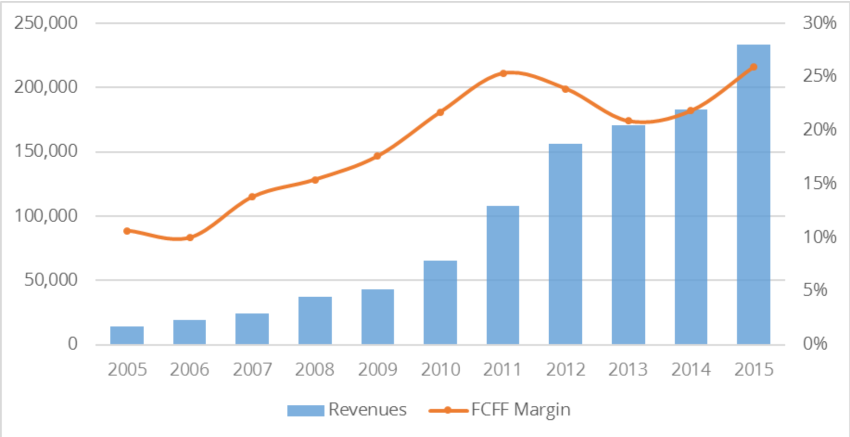

Figure 5. Historical revenue and FCFF margin for Apple

The consensus estimates for Apple's revenues imply growth rates of -8%, +4.9% and +5.6% for 2016, 2017 and 2018, respectively. The standard deviation for the past 11 years was 14.4%. The following chart shows Apple's historical growth rates and the future estimates, and also illustrates how in the model, the revenue growth rate for the estimates gradually declines from the growth rate the consensus estimates imply to the chosen perpetuity growth rate.

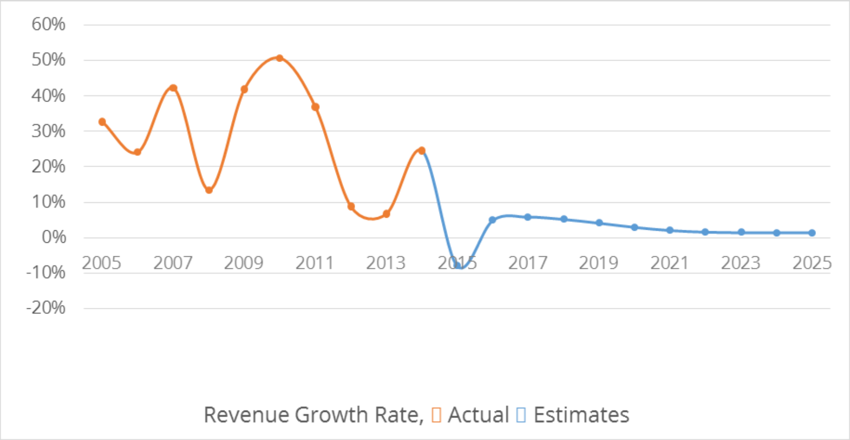

Figure 6. Historical and estimated revenue growth rates for Apple

Historically, the average cost of goods sold was 63% of revenues with a standard deviation of 4.6%, operating expenses were 12% with a standard deviation of 3%, the tax rate was 27.5% with a standard deviation of 2.7%, depreciation and amortization were 2.3% with a standard deviation of 1.3%, capital expenditures were 3.7% with a standard deviation of 1.1% and changes in net working capital were -2% with a standard deviation of 1.3%. These inputs are summarized in the following table.

Table 1. Inputs used in the valuation of Apple

Inputs	**Mean**	**Standard Deviation**
Revenue Growth Rate		
2016	-8.1 %	14.4 %
2017	4.9 %	14.4 %
2018	5.8 %	14.4 %
Cost of goods sold as % of revenue	63.0 %	4.6 %
Operating expenses as % of revenue	12.0 %	3.0 %
Tax Rate	27.5 %	2.7 %
D&A as % of revenue	2.3 %	1.3 %
Capital expenditure as % of revenue	3.7 %	1.1 %
Changes in net working capital as % of revenue	-2.0 %	1.3 %
WACC	11.1 %	1.5 %

The WACC has been on average 11.1% with a standard deviation of 1.5%. For the terminal growth rate, I used the average growth rate of the US GDP for the past 11 years which is 1.35%. The following graphs demonstrate the historical figures for these inputs. For the WACC, in order to give a sense of context for the figures chosen for the model, I also graphed the mean for the 11 years (the dotted line) and the shades of blue in the background, each shade representing the standard deviations 1-3 around this mean.

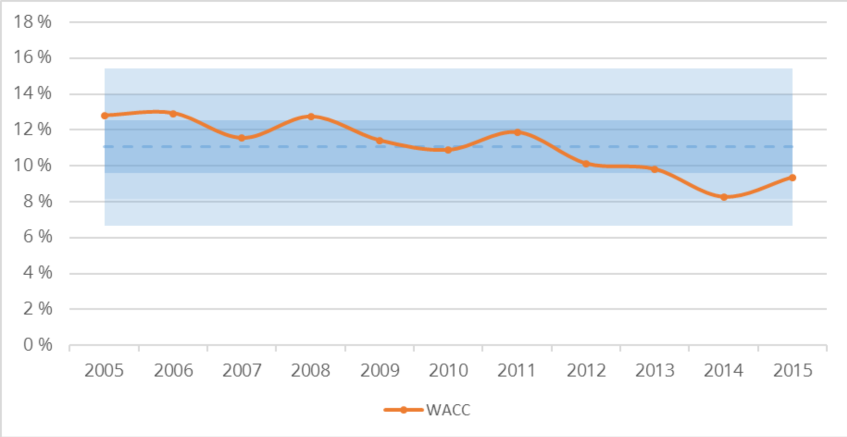

Figure 7. Historical WACC for Apple

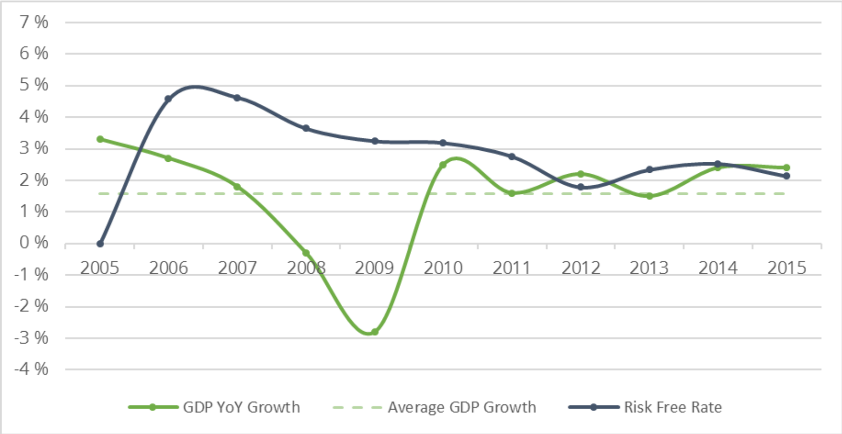

Figure 8. Historical GDP growth rate and risk-free rate

To demonstrate the behavior of the simulations and how they deviate around the mean, I created a graph showing the simulated growth rates for the first four iterations and the mean value. The graph illustrates well how they all follow approximately the same trend dictated by the mean values.

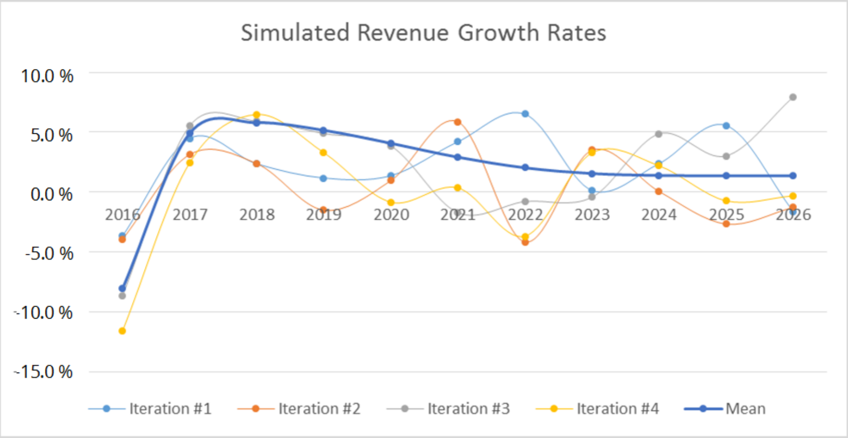

Figure 9. Mean estimated figures and simulated figures

Once the simulations and valuation was completed I arrived at the following results:

Table 2. Results of the valuation of Apple

APPLE INC

Ticker	AAPL US Equity
Currency	USD
Current stock price	96.68
Iterations	100,000
Base case implied upside	28.6 %
Base case estimated value	124.32
Simulation implied upside	17.9 %
Simulation median estimated value	114.00
Iterations above current price	93 %
Standard deviation	16 %
Upside to standard deviation ratio	1.10

After 100 000 iterations with the current stock price being 96.68 USD, the model suggested that Apple was, in fact, undervalued and that the mean value was approximately 114 USD, which implies an upside of 17.9%, with a standard deviation of 16%. Of all the iterations, 93% were above the current market price. When we compare the results between different valuations, we not only want to know the upside implied by the estimate for the most probable return, but also the level of certainty of this estimate. Therefore, the ratio between the upside implied by the mean and the standard deviation of the simulated stock values can be used for comparison between stocks. For Apple, this ratio was 17.9% / 16% = 1.1.

For a more complete representation of the results, I grouped the simulated values per share into ranges, and instances in every range were counted and divided by the number of iterations. This then gives the probability that the intrinsic value of the stock is within this range. The chart below shows the results for Apple

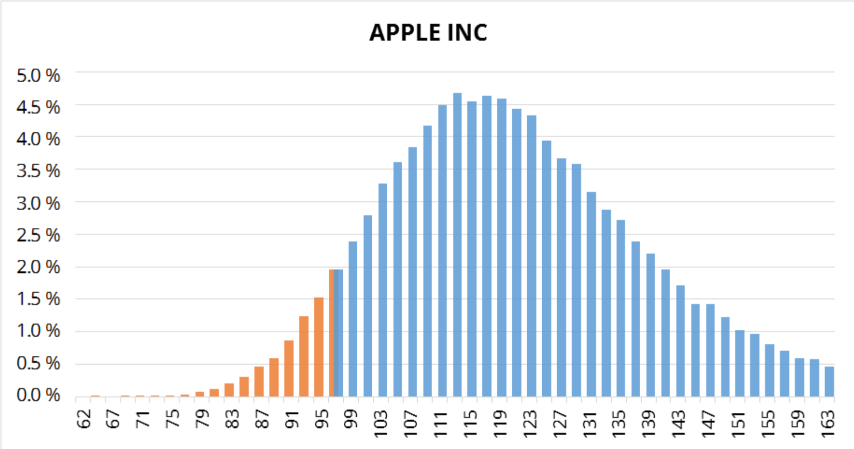

Figure 10. Simulation results for Apple

3.2.2 Microsoft

Microsoft's revenue grew from 39.8 billion USD in 2005 to 93.6 billion USD in 2015. For the same time period, the FCFF margin decreased from 29.3% to 21.6%.

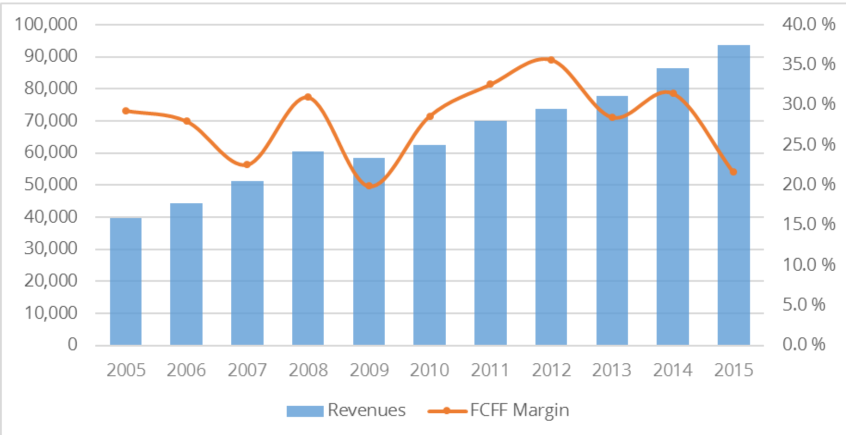

Figure 11. Historical revenue and FCFF margin for Microsoft

The consensus estimates for Microsoft's revenues implied growth rates of -2.6%, +5.0% and +7.6% for 2016, 2017 and 2018, respectively. The standard deviation for the past 11 years was 5.3%. The WACC was on average 9.9% with a standard deviation of 0.8%.

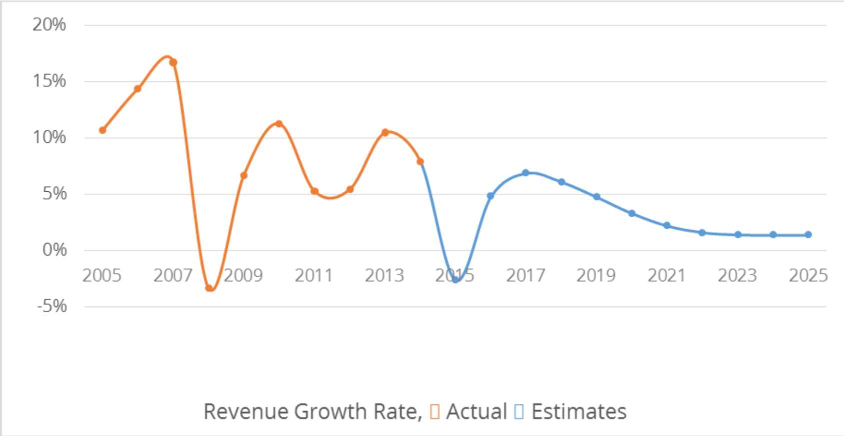

Figure 12. Historical and estimated revenue growth rates for Microsoft

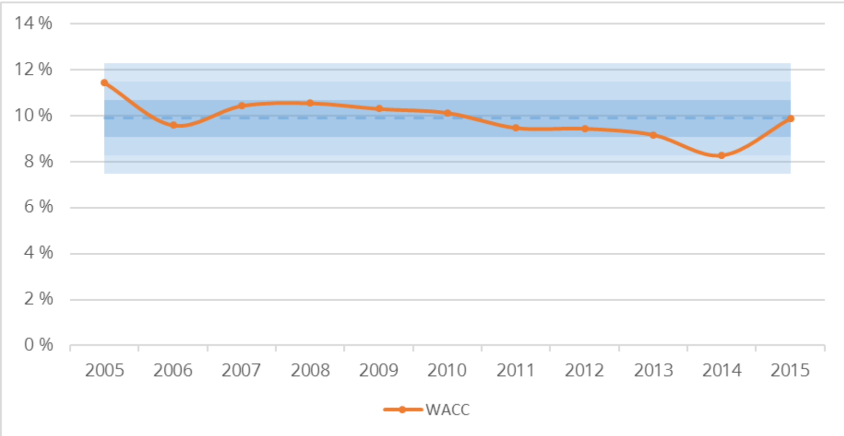

Figure 13. Historical WACC for Microsoft

For Microsoft, on average, the cost of goods sold was 22.9% of revenues with a standard deviation of 5.7%, operating expenses were 41.1% with a standard deviation of 3.8%, the tax rate was 24.1% with a standard deviation of 4.4%, depreciation and amortization were 4% with a standard deviation of 1.3%, capital expenditures were 4.4% with a standard deviation of 1.4% and changes in net working capital were -1.2% with a standard deviation of 3.2%. These inputs are summarized in the following table.

Table 3. Inputs used in the valuation of Microsoft

Inputs	Mean	Standard Deviation
Revenue Growth Rate		
2016	-2.8 %	5.3 %
2017	5.0 %	5.3 %
2018	7.6 %	5.3 %
Cost of goods sold as % of revenue	22.9 %	5.7 %
Operating expenses as % of revenue	41.1 %	3.8 %
Tax Rate	24.1 %	4.4 %
D&A as % of revenue	4.0 %	1.3 %
Capital expenditure as % of revenue	4.4 %	1.4 %
Changes in net working capital as % of revenue	-1.2 %	3.2 %
WACC	9.9 %	0.8 %

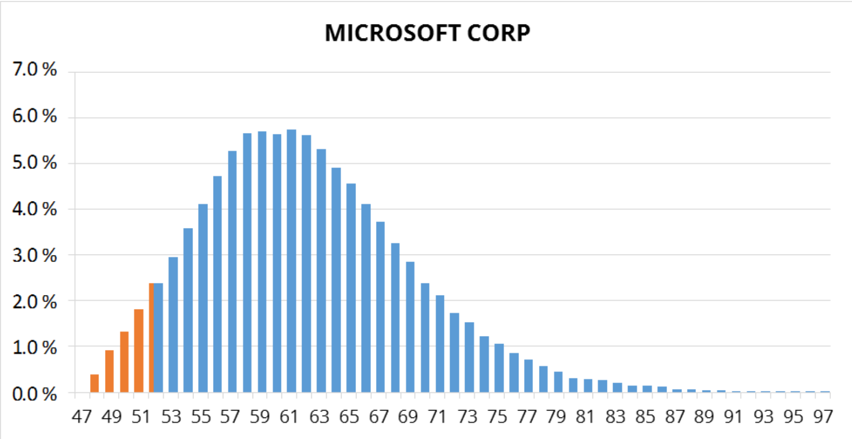

Figure 14. Simulation results for Microsoft

After 100 000 iterations with the current stock price being 51.16 USD, the mean value of the simulations was 61.00 USD, which implies an upside of +19.2%, with a standard deviation of 12%. Of all the iterations, 94% were above the current market price. The ratio between the upside and uncertainty for Microsoft was 1.62.

Table 4. Results of the valuation of Microsoft

MICROSOFT CORP

Ticker	MSFT US Equity
Currency	USD
Current stock price	51.16
Iterations	100,000
Base case implied upside	25.3 %
Base case estimated value	64.11
Simulation implied upside	19.2 %
Simulation median estimated value	61.00
Iterations above current price	94 %
Standard deviation	12 %
Upside to standard deviation ratio	1.62

3.2.3 IBM

IBM's revenue actually decreased from 91.1 billion USD in 2005 to 81.7 billion USD in 2015. For the same time period, the FCFF margin grew from 10.6% to 25.9%.

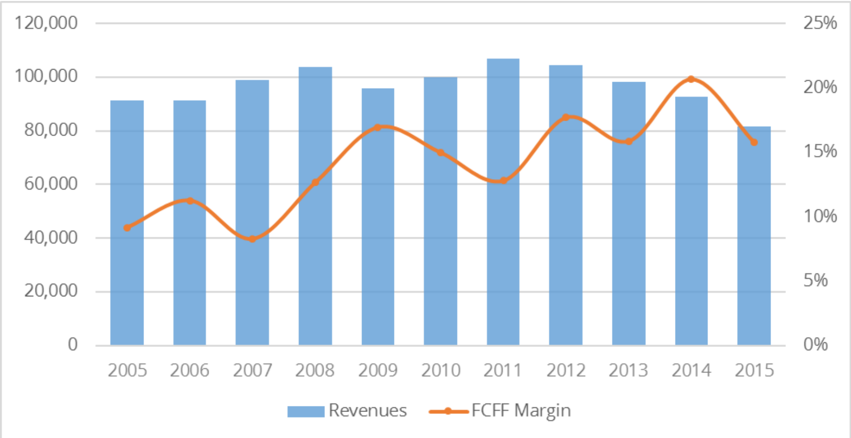

Figure 15. Historical revenue and FCFF margin for IBM

The consensus estimates for IBM's revenues implied growth rates of -2.6%, +5.0% and +7.6% for 2016, 2017 and 2018, respectively. The standard deviation for the past 11 years was 5.3%. The WACC was on average 8.5% with a standard deviation of 1.0%.

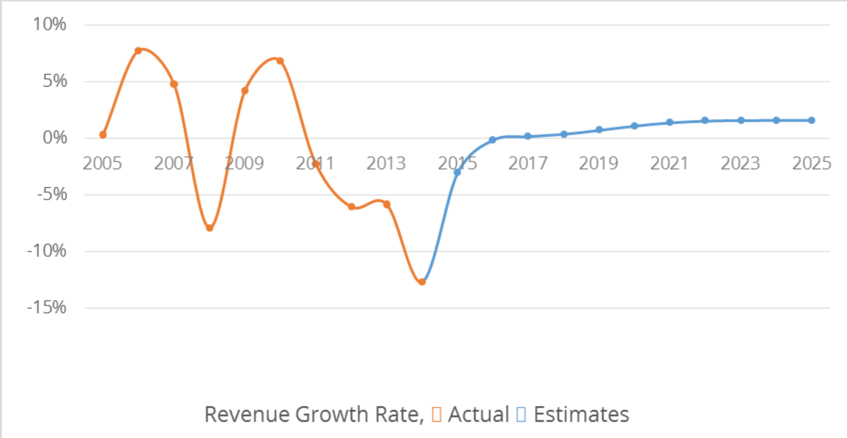

Figure 16. Historical and estimated revenue growth rates for IBM

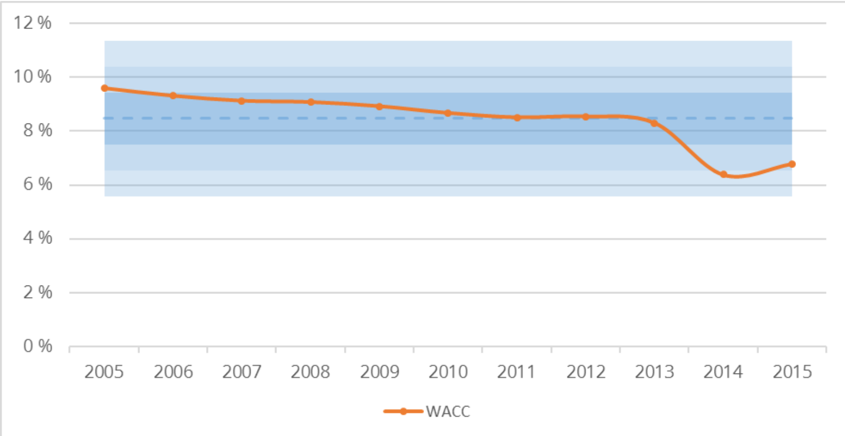

Figure 17. Historical WACC for IBM

For IBM, on average, the cost of goods sold were 54% of revenues with a standard deviation of 3.4%, operating expenses were 28.2% with a standard deviation of 0.8%, the tax rate was 25.2% with a standard deviation of 4.5%, depreciation and amortization were 5% with a standard deviation of 0.4%, capital expenditures were 4.1% with a standard deviation of 0.4% and changes in net working capital were 0.1% with a standard deviation of 1.8%. These inputs are summarized in the following table.

Table 5. Inputs used in the valuation of IBM

Inputs	**Mean**	**Standard Deviation**
Revenue Growth Rate		
2016	-3.0 %	6.6 %
2017	-0.2 %	6.6 %
2018	0.1 %	6.6 %
Cost of goods sold as % of revenue	54.0 %	3.4 %
Operating expenses as % of revenue	28.2 %	0.8 %
Tax Rate	25.2 %	4.5 %
D&A as % of revenue	5.0 %	0.4 %
Capital expenditure as % of revenue	4.1 %	0.4 %
Changes in net working capital as % of revenue	0.1 %	1.8 %
WACC	8.5 %	1.0 %

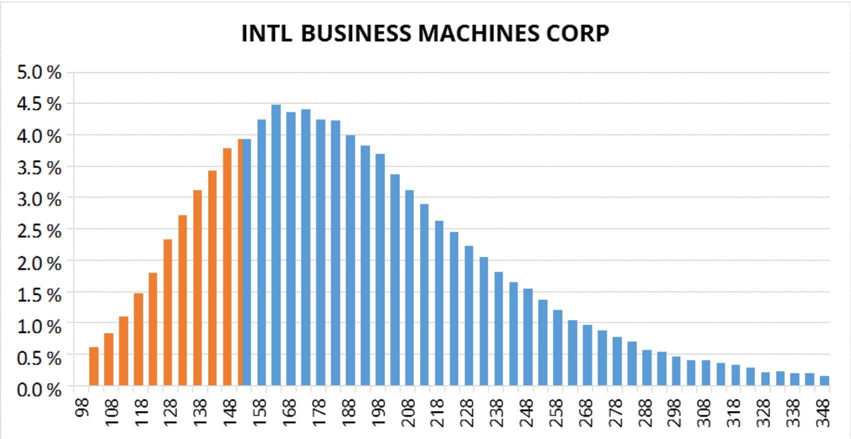

Figure 18. Simulation results for IBM

After 100 000 iterations with the current stock price being 152.35 USD, the mean value of the simulations was 189.00 USD, which implies an upside of +24.0%, with a standard deviation of 29%. Of all the iterations, 75% were above the current market price. The ratio between the upside and uncertainty for IBM was 0.82.

Table 6. Results of the valuation of IBM

INTL BUSINESS MACHINES CORP

Ticker	IBM US Equity
Currency	USD
Current stock price	152.35
Iterations	100,000
Base case implied upside	25.8 %
Base case estimated value	191.70
Simulation implied upside	24.0 %
Simulation mean estimated value	189.00
Iterations above current price	75 %
Standard deviation	29 %
Upside to standard deviation ratio	0.82

3.2.4 Results

The results implied that IBM had the highest upside at +24.0%, Microsoft the second highest at +19.2% and Apple the lowest at +17.9%. The standard deviations of the simulations implied that Microsoft had the least uncertainty related to its inputs at 12.0%, Apple the second least at 16.0% and IBM the most at 24.0%. By viewing the implied returns in relation to the level of uncertainty through their ratio, Microsoft offered the most return per level of uncertainty with a ratio of 1.62, Apple the second most at 1.10 and IBM the least at 0.82. The below table summarizes the results.

Table 7. Summary of the valuations of Apple, Microsoft and IBM

	Current stock price (USD)	Simulation mean estimated value	Simulation implied upside	Standard deviation	Upside to standard deviation ratio
Apple	96.7	114.0	17.9 %	16.0 %	1.10
Microsoft	51.16	61.0	19.2 %	12.0 %	1.62
IBM	152.4	189.0	24.0 %	29.0 %	0.82

The interesting aspect of these results was the fact that although IBM implied the highest upside, it also had the highest level of uncertainty. A more traditional approach to valuation would prioritize IBM over Microsoft and Apple based on the implied upside, and uncertainty wouldn’t be a factor in decision making. But through the added insight a simulation adds, the investor has a more complete picture and the valuation can be put into context with the uncertainty underlying the stock. A rational investor seeks the highest return-to-risk ratio and, given the results of this study, Microsoft would be the rational choice.

4 Conclusions

4.1 Summary

The goal of the study was to answer the research question of: can investment decisions be improved through the introduction of a probabilistic approach to valuation? By incorporating a simulation method to a discounted cash flow valuation model, the investor gains a more complete picture of the uncertainty related to the drivers of value for the asset. In order to answer the research question, I constructed a DCF model with the inputs defined as probability distributions, instead of single point figures. To test the model, I conducted valuations on Apple, Microsoft and IBM, and compared the results.

The findings of this study were that by putting the valuation in context with the uncertainty related to the asset, the investment decision could be improved. By viewing the upside the valuation model implies in relation to the amount of uncertainty related to the models inputs, provides a new aspect to the decision making. The investor can view the results not only through the expected return but also through the level of uncertainty.

4.2 Validity and reliability

The model was built by accurately following the theories studied in the theoretical framework which sets the validity for the studies. Although not entirely a new concept, Monte Carlo simulation is still a fairly uncommon approach in equity valuation which meant proper material on it was challenging to find. A broader set of reference material provides new aspects and adds to the validity of the research.

The main drawback in reliability in the study is the fact that the model wasn't backtested, meaning it wasn't tested on accurate point-in-time historical data for a long enough period of time to give an approximation of the viability of the model. The results of this study, although encouraging, could be a result of single instances. If repeated for a longer time period they could turn out to be inaccurate. However, conducting valuations on more than one company somewhat mitigates this risk.

The technical implementation is also prone to errors. The data was provided through the Bloomberg terminal and errors in the data are possible. The nature of a simulation means that there are a large number of steps in the process and it would be impossible to examine every single step. This could lead to errors in the code or data going undiscovered. Also, the assumption for the inputs of the simulation was that they are normally distributed, which in some cases might not hold true.

References

Benninga, S. 2014. Financial Modeling, Fourth Edition, The MIT Press, Cambridge.

Brandimarte, P. 2014. Handbook in Monte Carlo Simulation, Applications in Financial Engineering, Risk Management and Economics, John Wiley & Sons, Inc., New Jersey.

Damodaran, A. 2002a. Investment Valuation, 2nd Edition, John Wiley & Sons, Inc., New York.

Damodaran, A. 2003b. Value and Risk: Beyond Betas, URL: http://pages.stern.nyu.edu/~adamodar/

Damodaran, A. 2012c. The Dark Side of Valuation: Bias, Uncertainty and Complexity, URL: http://people.stern.nyu.edu/adamodar/pdfiles/country/ValuationBermudaTriangle.pdf

Damodaran, A. n.d. Probabilistic Approaches: Scenario Analysis, Decision Trees and Simulations, URL: http://pages.stern.nyu.edu/~adamodar/

French, N. & Gabrielli, L. 2005. Discounted Cash Flow: Accounting for Uncertainty

Pignataro, P. 2013. Financial Modeling and Valuation, John Wiley & Sons, Inc., New Jersey.

Pinto, J., Henry, E., Robinson,T., Stowe, J. & Rath, D. 2010. Equity Asset Valuation, 2nd Edition, John Wiley & Sons, Inc., New Jersey.

Viebig, J., Poddig, T. & Varmaz, A. 2010. Equity Valuation: Models from Leading Investment Banks, John Wiley & Sons, Inc., New Jersey.

Appendix 1 Apple FCFF Calculations, Estimates and Valuation

	-10FY	-9FY	-8FY	-7FY	-6FY	-5FY	-4FY	-3FY	-2FY	-1FY	-0FY
	30.9.2005	30.9.2006	30.9.2007	30.9.2008	30.9.2009	30.9.2010	30.9.2011	30.9.2012	30.9.2013	30.9.2014	30.9.2015
Revenue	**13,931**	**19,315**	**24,578**	**37,491**	**42,905**	**65,225**	**108,249**	**156,508**	**170,910**	**182,795**	**233,715**
% YoY Growth		32.7 %	24.1 %	42.2 %	13.5 %	41.9 %	50.7 %	36.9 %	8.8 %	6.7 %	24.6 %
(-) Cost of Revenue	9,889	13,717	16,426	24,294	25,683	39,541	64,431	87,846	106,606	112,258	140,089
% of Revenue	71.0 %	71.0 %	66.8 %	64.8 %	59.9 %	60.6 %	59.5 %	56.1 %	62.4 %	61.4 %	59.9 %
(=) Gross Profit	**4,042**	**5,598**	**8,152**	**13,197**	**17,222**	**25,684**	**43,818**	**68,662**	**64,304**	**70,537**	**93,626**
% Margin	29.0 %	29.0 %	33.2 %	35.2 %	40.1 %	39.4 %	40.5 %	43.9 %	37.6 %	38.6 %	40.1 %
(-) Operating Expenses/Income	2,399	3,145	3,745	4,870	5,482	7,299	10,028	13,421	15,305	18,034	22,396
% of Revenue	17.2 %	16.3 %	15.2 %	13.0 %	12.8 %	11.2 %	9.3 %	8.6 %	9.0 %	9.9 %	9.6 %
(=) Operating Income	**1,643**	**2,453**	**4,407**	**8,327**	**11,740**	**18,385**	**33,790**	**55,241**	**48,999**	**52,503**	**71,230**
% Margin	12 %	13 %	18 %	22 %	27 %	28 %	31 %	35 %	29 %	29 %	30 %
(-) Tax on Operating Income	436	722	1330	2632	3727	4489	8183	13899	12816	13735	18790
% Tax Rate	26.5 %	29.4 %	30.2 %	31.6 %	31.8 %	24.4 %	24.2 %	25.2 %	26.2 %	26.2 %	26.4 %
(=) NOPAT	**1,207**	**1,731**	**3,077**	**5,695**	**8,013**	**13,896**	**25,607**	**41,342**	**36,183**	**38,768**	**52,440**
% Margin	9 %	9 %	13 %	15 %	19 %	21 %	24 %	26 %	21 %	21 %	22 %
(+) Depreciation & Amortization	179	225	327	496	734	1,027	1,814	3,277	6,757	7,946	11,257
% of Revenue	1.3 %	1.2 %	1.3 %	1.3 %	1.7 %	1.6 %	1.7 %	2.1 %	4.0 %	4.3 %	4.8 %
(-) Capital Expenditure	260	657	735	1,091	1,144	2,005	4,260	8,295	8,165	9,571	11,247
% of Revenue	1.9 %	3.4 %	3.0 %	2.9 %	2.7 %	3.1 %	3.9 %	5.3 %	4.8 %	5.2 %	4.8 %
(-) Changes in Net Working Capital	-356	-627	-719	-675	51	-1,249	-4,270	-1,084	-900	-2,768	-8,148
% of Revenue	-2.6 %	-3.2 %	-2.9 %	-1.8 %	0.1 %	-1.9 %	-3.9 %	-0.7 %	-0.5 %	-1.5 %	-3.5 %
(=) Free Cash Flow	**1,482**	**1,926**	**3,388**	**5,775**	**7,552**	**14,167**	**27,431**	**37,408**	**35,675**	**39,911**	**60,598**
% Margin	10.6 %	10.0 %	13.8 %	15.4 %	17.6 %	21.7 %	25.3 %	23.9 %	20.9 %	21.8 %	25.9 %

	30.9.2016	30.9.2017	30.9.2018	30.9.2019	30.9.2020	30.9.2021	30.9.2022	30.9.2023	30.9.2024	30.9.2025	30.9.2026
Revenue	**214,852**	**225,371**	**238,414**	**250,702**	**260,901**	**268,481**	**273,903**	**278,133**	**281,976**	**285,795**	**289,666**
% YoY Growth	-8.1 %	4.9 %	5.8 %	5.2 %	4.1 %	2.9 %	2.0 %	1.5 %	1.4 %	1.4 %	1.4 %
(-) Cost of Revenue	135,453	142,085	150,308	158,055	164,485	169,264	172,682	175,349	177,771	180,179	182,620
% of Revenue	63.0 %	63.0 %	63.0 %	63.0 %	63.0 %	63.0 %	63.0 %	63.0 %	63.0 %	63.0 %	63.0 %
(=) Gross Profit	**79,399**	**83,286**	**88,106**	**92,647**	**96,416**	**99,217**	**101,221**	**102,784**	**104,204**	**105,616**	**107,046**
% Margin	37.0 %	37.0 %	37.0 %	37.0 %	37.0 %	37.0 %	37.0 %	37.0 %	37.0 %	37.0 %	37.0 %
(-) Operating Expenses/Income	25,771	27,032	28,597	30,071	31,294	32,203	32,853	33,361	33,822	34,280	34,744
% of Revenue	12.0 %	12.0 %	12.0 %	12.0 %	12.0 %	12.0 %	12.0 %	12.0 %	12.0 %	12.0 %	12.0 %
(=) Operating Income	**53,628**	**56,254**	**59,509**	**62,576**	**65,122**	**67,014**	**68,367**	**69,423**	**70,382**	**71,336**	**72,302**
% Margin	25.0 %	25.0 %	25.0 %	25.0 %	25.0 %	25.0 %	25.0 %	25.0 %	25.0 %	25.0 %	25.0 %
(-) Tax on Operating Income	14723	15444	16338	17180	17879	18398	18770	19060	19323	19585	19850
% Tax Rate	27.5 %	27.5 %	27.5 %	27.5 %	27.5 %	27.5 %	27.5 %	27.5 %	27.5 %	27.5 %	27.5 %
(=) NOPAT	**38,905**	**40,810**	**43,171**	**45,396**	**47,243**	**48,616**	**49,598**	**50,364**	**51,059**	**51,751**	**52,452**
% Margin	18.1 %	18.1 %	18.1 %	18.1 %	18.1 %	18.1 %	18.1 %	18.1 %	18.1 %	18.1 %	18.1 %
(+) Depreciation & Amortization	4,937	5,178	5,478	5,760	5,995	6,169	6,294	6,391	6,479	6,567	6,656
% of Revenue	2.3 %	2.3 %	2.3 %	2.3 %	2.3 %	2.3 %	2.3 %	2.3 %	2.3 %	2.3 %	2.3 %
(-) Capital Expenditure	8,002	8,394	8,880	9,337	9,717	10,000	10,202	10,359	10,502	10,644	10,789
% of Revenue	3.7 %	3.7 %	3.7 %	3.7 %	3.7 %	3.7 %	3.7 %	3.7 %	3.7 %	3.7 %	3.7 %
(-) Changes in Net Working Capital	-4,392	-4,607	-4,874	-5,125	-5,334	-5,489	-5,600	-5,686	-5,765	-5,843	-5,922
% of Revenue	-2.0 %	-2.0 %	-2.0 %	-2.0 %	-2.0 %	-2.0 %	-2.0 %	-2.0 %	-2.0 %	-2.0 %	-2.0 %
(=) Free Cash Flow	**40,232**	**42,201**	**44,644**	**46,945**	**48,855**	**50,274**	**51,289**	**52,081**	**52,801**	**53,516**	**54,241**
% Margin	18.7 %	18.7 %	18.7 %	18.7 %	18.7 %	18.7 %	18.7 %	18.7 %	18.7 %	18.7 %	18.7 %
Present Value of Free Cash Flows	**39,755**	**39,100**	**37,242**	**35,261**	**33,030**	**30,604**	**28,111**	**25,702**	**23,454**	**21,404**	**19,767**

Free Cash Flow at Year 11	54,241
WACC	11.06 %
Perpetuity Growth Rate	1.35 %
Perpetuity Value at End of Year 11	566,200
Present Perpetuity Value	206,341
Present Value of FCF	333,430
(=) Enterprise Value	**539,771**
(+) Short Term Debt	10,999
(+) Long Term Debt	53,463
(-) Cash and Marketable Securities	205,666
(=) Net Debt	**-141,204**
(-) Current Preferred and Minority Interest	0
(=) Equity Value	**680,975**
Shares Outstanding	5,477
Estimated Value per Share	**124.32**
Current Price	96.68
Estimated Upside	28.6 %

Appendix 2 Microsoft FCFF Calculations, Estimates and Valuation

	-10FY 30.6.2005	-9FY 30.6.2006	-8FY 30.6.2007	-7FY 30.6.2008	-6FY 30.6.2009	-5FY 30.6.2010	-4FY 30.6.2011	-3FY 30.6.2012	-2FY 30.6.2013	-1FY 30.6.2014	-0FY 30.6.2015
Revenue	**39,788**	**44,282**	**51,122**	**60,420**	**58,437**	**62,484**	**69,943**	**73,723**	**77,849**	**86,451**	**93,580**
% YoY Growth		10.7 %	14.4 %	16.7 %	-3.3 %	6.7 %	11.3 %	5.3 %	5.4 %	10.5 %	7.9 %
(-) Cost of Revenue	6,200	7,650	10,693	11,598	12,155	12,395	15,577	17,530	20,249	27,078	33,038
% of Revenue	15.6 %	17.3 %	20.9 %	19.2 %	20.8 %	19.8 %	22.3 %	23.8 %	26.0 %	31.3 %	35.3 %
(=) Gross Profit	**33,588**	**36,632**	**40,429**	**48,822**	**46,282**	**50,089**	**54,366**	**56,193**	**57,600**	**59,373**	**60,542**
% Margin	84.4 %	82.7 %	79.1 %	80.8 %	79.2 %	80.2 %	77.7 %	76.2 %	74.0 %	68.7 %	64.7 %
(-) Operating Expenses/Income	19,027	20,160	21,905	26,551	25,306	25,932	27,205	28,237	30,103	31,869	32,403
% of Revenue	47.8 %	45.5 %	42.8 %	43.9 %	43.3 %	41.5 %	38.9 %	38.3 %	38.7 %	36.9 %	34.6 %
(=) Operating Income	**14,561**	**16,472**	**18,524**	**22,271**	**20,976**	**24,157**	**27,161**	**27,956**	**27,497**	**27,504**	**28,139**
% Margin	37 %	37 %	36 %	37 %	36 %	39 %	39 %	38 %	35 %	32 %	30 %
(-) Tax on Operating Income	3830	5108	5562	5736	5691	6018	5152	5103	5371	5116	7024
% Tax Rate	26.3 %	31.0 %	30.0 %	25.8 %	27.1 %	24.9 %	19.0 %	18.3 %	19.5 %	18.6 %	25.0 %
(=) NOPAT	**10,731**	**11,364**	**12,962**	**16,535**	**15,285**	**18,139**	**22,009**	**22,853**	**22,126**	**22,388**	**21,115**
% Margin	27 %	26 %	25 %	27 %	26 %	29 %	31 %	31 %	28 %	26 %	23 %
(+) Depreciation & Amortization	855	903	1,440	2,056	2,562	2,673	2,766	2,967	3,755	5,212	5,957
% of Revenue	2.1 %	2.0 %	2.8 %	3.4 %	4.4 %	4.3 %	4.0 %	4.0 %	4.8 %	6.0 %	6.4 %
(-) Capital Expenditure	812	1,578	2,264	3,182	3,119	1,977	2,355	2,305	4,257	5,485	5,944
% of Revenue	2.0 %	3.6 %	4.4 %	5.3 %	5.3 %	3.2 %	3.4 %	3.1 %	5.5 %	6.3 %	6.4 %
(-) Changes in Net Working Capital	-890	-1,708	596	-3,309	3,105	942	-369	-2,785	-561	-5,114	900
% of Revenue	-2.2 %	-3.9 %	1.2 %	-5.5 %	5.3 %	1.5 %	-0.5 %	-3.8 %	-0.7 %	-5.9 %	1.0 %
(=) Free Cash Flow	**11,664**	**12,397**	**11,542**	**18,718**	**11,623**	**17,893**	**22,789**	**26,300**	**22,185**	**27,229**	**20,228**
% Margin	29.3 %	28.0 %	22.6 %	31.0 %	19.9 %	28.6 %	32.6 %	35.7 %	28.5 %	31.5 %	21.6 %

	30.6.2016	30.6.2017	30.6.2018	30.6.2019	30.6.2020	30.6.2021	30.6.2022	30.6.2023	30.6.2024	30.6.2025	30.6.2026
Revenue	**90,925**	**95,503**	**102,721**	**109,587**	**115,271**	**119,401**	**122,220**	**124,305**	**126,147**	**127,969**	**129,817**
% YoY Growth	-2.8 %	5.0 %	7.6 %	6.7 %	5.2 %	3.6 %	2.4 %	1.7 %	1.5 %	1.4 %	1.4 %
(-) Cost of Revenue	20,854	21,904	23,560	25,135	26,438	27,386	28,032	28,510	28,933	29,351	29,774
% of Revenue	22.9 %	22.9 %	22.9 %	22.9 %	22.9 %	22.9 %	22.9 %	22.9 %	22.9 %	22.9 %	22.9 %
(=) Gross Profit	**70,071**	**73,598**	**79,161**	**84,452**	**88,833**	**92,016**	**94,188**	**95,795**	**97,214**	**98,618**	**100,042**
% Margin	77.1 %	77.1 %	77.1 %	77.1 %	77.1 %	77.1 %	77.1 %	77.1 %	77.1 %	77.1 %	77.1 %
(-) Operating Expenses/Income	37,387	39,269	42,237	45,060	47,398	49,096	50,255	51,112	51,870	52,619	53,378
% of Revenue	41.1 %	41.1 %	41.1 %	41.1 %	41.1 %	41.1 %	41.1 %	41.1 %	41.1 %	41.1 %	41.1 %
(=) Operating Income	**32,684**	**34,329**	**36,924**	**39,392**	**41,435**	**42,920**	**43,933**	**44,683**	**45,345**	**45,999**	**46,664**
% Margin	35.9 %	35.9 %	35.9 %	35.9 %	35.9 %	35.9 %	35.9 %	35.9 %	35.9 %	35.9 %	35.9 %
(-) Tax on Operating Income	7887	8284	8911	9506	9999	10358	10602	10783	10943	11101	11261
% Tax Rate	24.1 %	24.1 %	24.1 %	24.1 %	24.1 %	24.1 %	24.1 %	24.1 %	24.1 %	24.1 %	24.1 %
(=) NOPAT	**24,796**	**26,045**	**28,013**	**29,886**	**31,436**	**32,562**	**33,331**	**33,900**	**34,402**	**34,899**	**35,403**
% Margin	27.3 %	27.3 %	27.3 %	27.3 %	27.3 %	27.3 %	27.3 %	27.3 %	27.3 %	27.3 %	27.3 %
(+) Depreciation & Amortization	3,659	3,843	4,134	4,410	4,639	4,805	4,918	5,002	5,076	5,150	5,224
% of Revenue	4.0 %	4.0 %	4.0 %	4.0 %	4.0 %	4.0 %	4.0 %	4.0 %	4.0 %	4.0 %	4.0 %
(-) Capital Expenditure	4,006	4,207	4,525	4,828	5,078	5,260	5,384	5,476	5,557	5,637	5,719
% of Revenue	4.4 %	4.4 %	4.4 %	4.4 %	4.4 %	4.4 %	4.4 %	4.4 %	4.4 %	4.4 %	4.4 %
(-) Changes in Net Working Capital	-1,121	-1,178	-1,267	-1,351	-1,421	-1,472	-1,507	-1,533	-1,555	-1,578	-1,601
% of Revenue	-1.2 %	-1.2 %	-1.2 %	-1.2 %	-1.2 %	-1.2 %	-1.2 %	-1.2 %	-1.2 %	-1.2 %	-1.2 %
(=) Free Cash Flow	**25,571**	**26,858**	**28,888**	**30,819**	**32,418**	**33,580**	**34,372**	**34,959**	**35,477**	**35,989**	**36,509**
% Margin	28.1 %	28.1 %	28.1 %	28.1 %	28.1 %	28.1 %	28.1 %	28.1 %	28.1 %	28.1 %	28.1 %
Present Value of Free Cash Flows	**25,583**	**25,718**	**25,312**	**24,711**	**23,779**	**22,539**	**21,112**	**19,649**	**18,242**	**16,934**	**15,712**

Free Cash Flow at Year 11	36,509
WACC	9.3 %
Perpetuity Growth Rate	1.4 %
Perpetuity Value at End of Year 11	472,592
Present Perpetuity Value	203,384
Present Value of FCF	239,291
(=) Enterprise Value	**442,676**
(+) Short Term Debt	7,484
(+) Long Term Debt	27,808
(-) Cash and Marketable Securities	96,526
(=) Net Debt	**-61,234**
(-) Current Preferred and Minority Interest	0
(=) Equity Value	**503,910**
Shares Outstanding	7,860
Estimated Value per Share	**64.11**
Current Price	51.16
Estimated Upside	25.3 %

Appendix 3 IBM FCFF Calculations, Estimates and Valuation

	31.12.2005	31.12.2006	31.12.2007	31.12.2008	31.12.2009	31.12.2010	31.12.2011	31.12.2012	31.12.2013	31.12.2014	31.12.2015
Revenue	**91,134**	**91,424**	**98,786**	**103,630**	**95,757**	**99,870**	**106,916**	**104,507**	**98,367**	**92,793**	**81,741**
% YoY Growth		0.3 %	7.7 %	4.8 %	-7.9 %	4.2 %	6.8 %	-2.3 %	-6.1 %	-5.8 %	-12.7 %
(-) Cost of Revenue	54,602	53,129	57,057	57,969	51,973	54,061	56,776	53,945	49,054	46,213	40,588
% of Revenue	59.9 %	58.1 %	57.8 %	55.9 %	54.3 %	54.1 %	53.1 %	51.6 %	49.9 %	49.8 %	49.7 %
(=) Gross Profit	**36,532**	**38,295**	**41,729**	**45,661**	**43,784**	**45,809**	**50,140**	**50,562**	**49,313**	**46,580**	**41,153**
% Margin	40.1 %	41.9 %	42.2 %	44.1 %	45.7 %	45.9 %	46.9 %	48.4 %	50.1 %	50.2 %	50.3 %
(-) Operating Expenses/Income	27,156	26,366	28,213	29,723	26,298	27,432	29,440	28,743	27,700	25,107	23,931
% of Revenue	29.8 %	28.8 %	28.6 %	28.7 %	27.5 %	27.5 %	27.5 %	27.5 %	28.2 %	27.1 %	29.3 %
(=) Operating Income	**9,376**	**11,929**	**13,516**	**15,938**	**17,486**	**18,377**	**20,700**	**21,819**	**21,613**	**21,473**	**17,222**
% Margin	10 %	13 %	14 %	15 %	18 %	18 %	19 %	21 %	22 %	23 %	21 %
(-) Tax on Operating Income	3245	3494	3798	4177	4568	4508	5078	5383	3927	4962	3142
% Tax Rate	34.6 %	29.3 %	28.1 %	26.2 %	26.1 %	24.5 %	24.5 %	24.7 %	18.2 %	23.1 %	18.2 %
(=) NOPAT	**6,131**	**8,435**	**9,718**	**11,761**	**12,918**	**13,869**	**15,622**	**16,436**	**17,686**	**16,511**	**14,080**
% Margin	7 %	9 %	10 %	11 %	13 %	14 %	15 %	16 %	18 %	18 %	17 %
(+) Depreciation & Amortization	5,188	4,983	5,201	5,450	4,994	4,831	4,815	4,676	4,678	4,492	3,855
% of Revenue	5.7 %	5.5 %	5.3 %	5.3 %	5.2 %	4.8 %	4.5 %	4.5 %	4.8 %	4.8 %	4.7 %
(-) Capital Expenditure	3,842	4,362	4,630	4,171	3,447	4,185	4,108	4,082	3,623	3,740	3,579
% of Revenue	4.2 %	4.8 %	4.7 %	4.0 %	3.6 %	4.2 %	3.8 %	3.9 %	3.7 %	4.0 %	4.4 %
(-) Changes in Net Working Capital	-847	-1,224	2,141	-58	-1,771	-446	2,664	-1,485	3,132	-1,941	1,451
% of Revenue	-0.9 %	-1.3 %	2.2 %	-0.1 %	-1.8 %	-0.4 %	2.5 %	-1.4 %	3.2 %	-2.1 %	1.8 %
(=) Free Cash Flow	**8,324**	**10,280**	**8,148**	**13,098**	**16,236**	**14,961**	**13,665**	**18,515**	**15,609**	**19,204**	**12,905**
% Margin	9.1 %	11.2 %	8.2 %	12.6 %	17.0 %	15.0 %	12.8 %	17.7 %	15.9 %	20.7 %	15.8 %

	31.12.2016	31.12.2017	31.12.2018	31.12.2019	31.12.2020	31.12.2021	31.12.2022	31.12.2023	31.12.2024	31.12.2025	31.12.2026
Revenue	**79,272**	**79,129**	**79,219**	**79,460**	**79,961**	**80,743**	**81,748**	**82,882**	**84,072**	**85,286**	**86,518**
% YoY Growth	-3.0 %	-0.2 %	0.1 %	0.3 %	0.6 %	1.0 %	1.2 %	1.4 %	1.4 %	1.4 %	1.4 %
(-) Cost of Revenue	42,820	42,742	42,791	42,921	43,192	43,614	44,157	44,770	45,413	46,068	46,734
% of Revenue	54.0 %	54.0 %	54.0 %	54.0 %	54.0 %	54.0 %	54.0 %	54.0 %	54.0 %	54.0 %	54.0 %
(=) Gross Profit	**36,452**	**36,386**	**36,428**	**36,539**	**36,769**	**37,129**	**37,591**	**38,112**	**38,660**	**39,218**	**39,784**
% Margin	46.0 %	46.0 %	46.0 %	46.0 %	46.0 %	46.0 %	46.0 %	46.0 %	46.0 %	46.0 %	46.0 %
(-) Operating Expenses/Income	22,365	22,325	22,350	22,418	22,559	22,780	23,064	23,383	23,719	24,062	24,409
% of Revenue	28.2 %	28.2 %	28.2 %	28.2 %	28.2 %	28.2 %	28.2 %	28.2 %	28.2 %	28.2 %	28.2 %
(=) Operating Income	**14,087**	**14,062**	**14,078**	**14,121**	**14,210**	**14,349**	**14,527**	**14,729**	**14,940**	**15,156**	**15,375**
% Margin	17.8 %	17.8 %	17.8 %	17.8 %	17.8 %	17.8 %	17.8 %	17.8 %	17.8 %	17.8 %	17.8 %
(-) Tax on Operating Income	3555	3549	3553	3564	3586	3621	3666	3717	3770	3825	3880
% Tax Rate	25.2 %	25.2 %	25.2 %	25.2 %	25.2 %	25.2 %	25.2 %	25.2 %	25.2 %	25.2 %	25.2 %
(=) NOPAT	**10,532**	**10,513**	**10,525**	**10,557**	**10,624**	**10,728**	**10,861**	**11,012**	**11,170**	**11,331**	**11,495**
% Margin	13.3 %	13.3 %	13.3 %	13.3 %	13.3 %	13.3 %	13.3 %	13.3 %	13.3 %	13.3 %	13.3 %
(+) Depreciation & Amortization	3,964	3,957	3,962	3,974	3,999	4,038	4,088	4,145	4,204	4,265	4,327
% of Revenue	5.0 %	5.0 %	5.0 %	5.0 %	5.0 %	5.0 %	5.0 %	5.0 %	5.0 %	5.0 %	5.0 %
(-) Capital Expenditure	3,267	3,261	3,264	3,274	3,295	3,327	3,369	3,415	3,464	3,515	3,565
% of Revenue	4.1 %	4.1 %	4.1 %	4.1 %	4.1 %	4.1 %	4.1 %	4.1 %	4.1 %	4.1 %	4.1 %
(-) Changes in Net Working Capital	107	107	107	107	108	109	110	112	114	115	117
% of Revenue	0.1 %	0.1 %	0.1 %	0.1 %	0.1 %	0.1 %	0.1 %	0.1 %	0.1 %	0.1 %	0.1 %
(=) Free Cash Flow	**11,123**	**11,103**	**11,115**	**11,149**	**11,219**	**11,329**	**11,470**	**11,629**	**11,796**	**11,967**	**12,140**
% Margin	14.0 %	14.0 %	14.0 %	14.0 %	14.0 %	14.0 %	14.0 %	14.0 %	14.0 %	14.0 %	14.0 %
Present Value of Free Cash Flows	**10,944**	**10,403**	**9,758**	**9,170**	**8,644**	**8,178**	**7,757**	**7,368**	**7,001**	**6,654**	**6,428**

Free Cash Flow at Year 11	12,140
WACC	6.7 %
Perpetuity Growth Rate	1.4 %
Perpetuity Value at End of Year 11	232,767
Present Perpetuity Value	123,255
Present Value of FCF	92,306
(=) Enterprise Value	**215,561**
(+) Short Term Debt	6,461
(+) Long Term Debt	33,428
(-) Cash and Marketable Securities	8,194
(=) Net Debt	**31,695**
(-) Current Preferred and Minority Interest	-162
(=) Equity Value	**184,028**
Shares Outstanding	960
Estimated Value per Share	**191.70**
Current Price	152.35
Estimated Upside	25.8 %