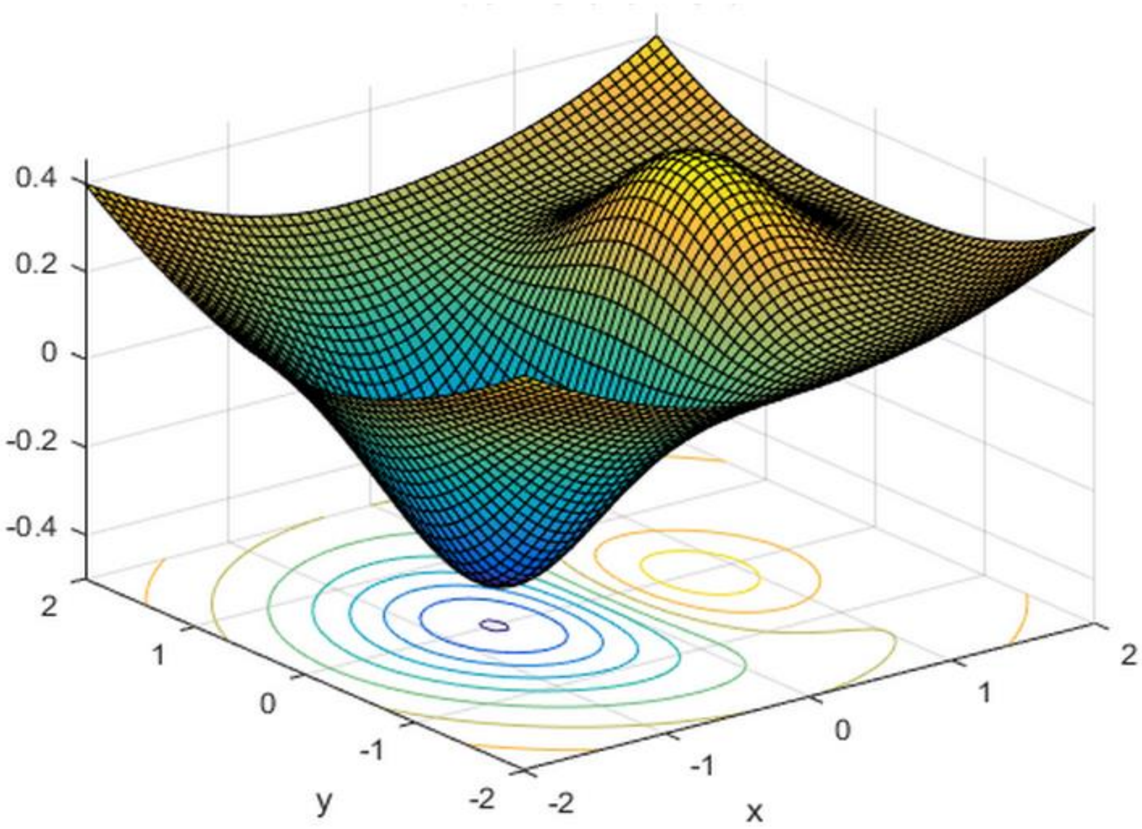
Savonia University of Applied Sciences

SAVONIA

# DEVELOPMENT OF ALGORITHM FOR LINEAR DISCRETE SYSTEM WITH QUADRATIC CRITERION OF QUALITY




16. January 2017

Akbota Yergali
Second year Master's degree of
Mechanics and mathematics faculty of
Al-Farabi Kazakh National University
Almaty Kazakstan

Harri Heikura
Principal Lecturer
Savonia University
of Applied Sciencies
Varkaus Finland

# DEVELOPMENT OF ALGORITHM FOR LINEAR DISCRETE SYSTEM WITH QUADRATIC CRITERION OF QUALITY

Discrete manufacturing is a type of production in which the original material (raw materials) during processing into the final product, goes through a finite number of technological and Assembly operations.

Usually the beginning and end of operations are defined by the signals of the two position sensors. A discrete type of production prevails in mechanical engineering, instrument making, light industry, enterprises for the production of furniture, packaging, pharmaceuticals, etc.

According to the International Association of engineers, such production exists more than 75% of industrial enterprises in the world. Even where the output is purely continuous, as are auxiliary discrete processes.

The discrete processes of management acquire an all greater value in a theory and practice of optimal management. It is related to that many tasks of the economic planning, technologies and organizations of production, are described by difference equalizations. In practice mostly and state information process, and management by a process it is come by true in discrete moments of time.

We will consider the process, described by next difference equalization:

$$x(k+1) = A(k)x(k) + B(k)u(k) - s(k) \quad (1)$$
$$(k = 0,1,\dots,N-1)$$
$$x(0) = a \quad (2)$$

with a limit management.

$$h_1(k) \le u(k) \le h_2(k), \quad (k = 0,1,\dots,N-1) \quad (3)$$

Let the criterion of quality look like :

$$J\big(x(k),u(k)\big) = \frac{1}{2}\big(x(N),Q(N)x(N)\big) + + \sum_{k=0}^{N-1}\left[\frac{1}{2}\big(x(k),Q(k)x(k)\big) + \big(a(k),x(k)\big)\right] + \sum_{k=0}^{N-1}\left[\frac{1}{2}\big(u(k),R(k)u(k)\big) + +\big(b(k),x(k)\big)\right] \quad (4)$$

The matrix of $Q(k)$ is considered symmetric and unpositively certain, and $R(k)$ symmetric and positively certain.

The problem is set: to find the management of $u(k)$ and trajectory of $x(k)$ satisfying to equalizations of motion (1) - (3) and delivering a minimum value to the criterion of quality (4).

For a decision the set problem we use Langrangian, after simple transformations, we will get the task of the quadratic dynamic programming.

Let

$$f(x,u) = A(k)x(k) + B(k)u(k) - s(k) \tag{5}$$

Lagrangian - L , where λ = Lagrange multiplier

$$L(x,u,\lambda) = f(x,u) - \lambda\big(J(x,u)\big) \tag{6}$$

$$L(x,u,\lambda) = A(k)x(k) + B(k)u(k) - s(k) - \lambda\Big\{\Big(\tfrac{1}{2}\big(x(N), Q(N)x(N)\big) + \\ + \textstyle\sum_{k=0}^{N-1}\Big[\tfrac{1}{2}\big(x(k), Q(k)x(k)\big) + \big(a(k), x(k)\big)\Big] + \sum_{k=0}^{N-1}\Big[\tfrac{1}{2}\big(u(k), R(k)u(k)\big) + \\ + \big(b(k), x(k)\big)\Big]\Big\} \tag{7}$$

After that this can

maximize $f(x,u)$
subject to $J(x,u) = 0$

After that we take gradient using

$$\Delta x, u \;\; f = \lambda \Delta x, u \;\; J \tag{8}$$

Where Δ partial derivate

$$\Delta x, u \;\; f = \left(\frac{\delta f}{x\delta}, \frac{\delta f}{u\delta}\right) \tag{9}$$

$$\Delta x, u \;\; J = \left(\frac{\delta J}{x\delta}, \frac{\delta J}{u\delta}\right) \tag{10}$$

Solve now

$$\Delta x, u, \lambda \;\; L(x,u,\lambda) = 0 \tag{11}$$

The purpose of the theory is to minimize quality problems in processes. The plant produces a certain number of products. Some of the products may occur defects, the amount of which should be minimized. Faults can be of varying size and their classification requires accurate measurements. By controlling the process properly using input variable $u(k)$ production line can be optimized for better quality. The result is a lot of high-quality products, represented by the variable $x(k+1)$.

Product quality criteria defined cost function $J\big(x(k), u(k)\big)$. Quality information is provided by the factory to develop a system. On the other hand made the customer queries directly from the market and the data is analyzed using dig data tools. Information about quality criteria to create $Q(k)$ matrix by scaling. $\mathrm{R}(k)$ matrix contains various types of quality factors.

Quadratic criterion of quality by applying the optimization method can be solved minimum quality errors.

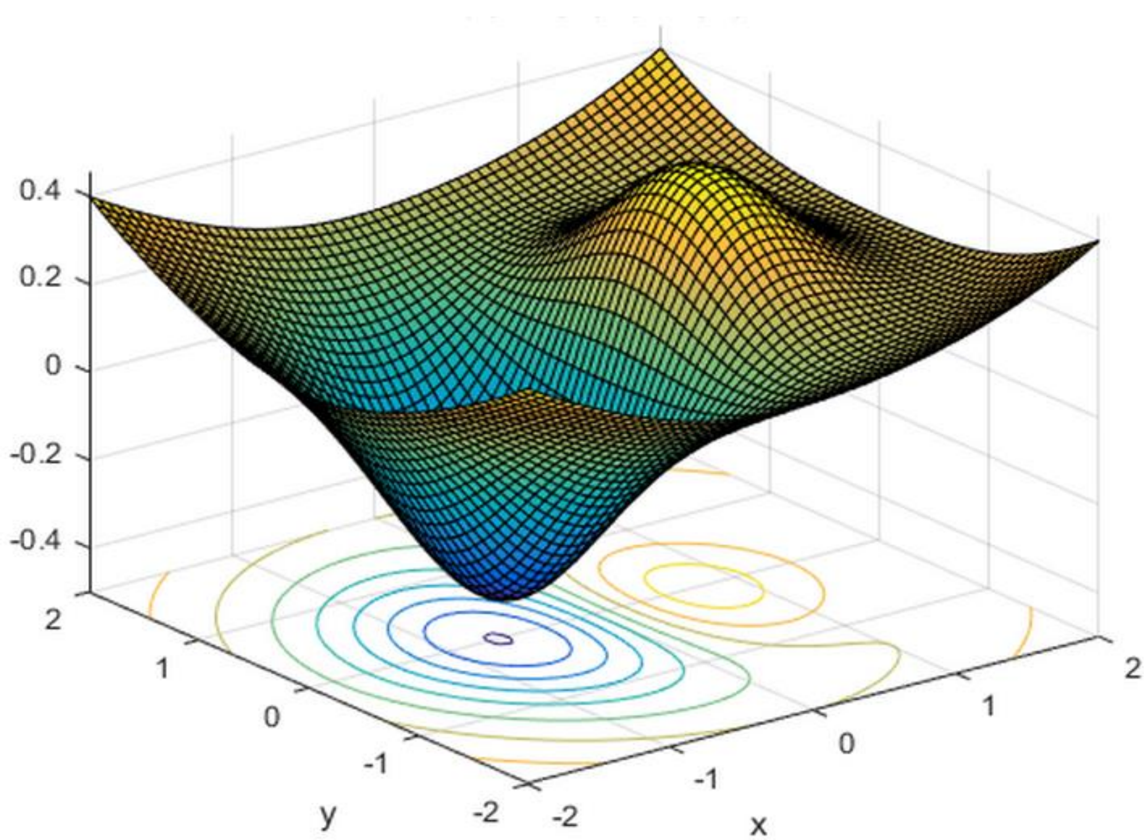



Figure 1. Optimization coordinates of function presented as surface, ( MathWorks )

The figure shows the minimum where the bugs are out, and the quality at its best.

The method can find application areas rich in targets which have several quality criteria after each other. At the same time the quality $q_1$ and $q_2$ defects as $y$ and $x$ in the picture. Optimization process gives a common effect as defined in 3 dimensional axis $z$. The task can be solved in several ways and there is today in Matlab the quadratic programming tools where we can build up minimize functions using Lagrangia principle. The constrains $h_1(k)$ and $h_2(k)$ must be determinated according limits of the tasks.

**References**

Chiang, Alpha C., *Fundamental Methods of Mathematical Economics*, McGraw-Hill, third edition, 1984: p. 386. ISBN 9757860069

G.F. Hadley, "Nonlinear and dynamic programming", Addison-Wesley (1964)

MathWorks, Accelerating the pace of engineering and science, 2017

Lasdon, Leon S. (2002). *Optimization theory for large systems* (reprint of the 1970 Macmillan ed.). Mineola, New York: Dover Publications, Inc. pp. xiii+523

*Vapnyarskii, I.B. (2001), "Lagrange multipliers", in Hazewinkel, Michiel, Encyclopedia of Mathematics, Springer, ISBN 978-1-55608-010-4*.