Applications of linear programming

Case study, minimizing the costs of transportation problem

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Abstract:

This research work represents transportation modeling approaches and forecasting techniques addressing the transportation flow of cargo containers with semi-processed goods on the selected routes from a certain number of suppliers with various production capacities to the certain points of destination. The aim is to achieve the minimum cost of transportation flow and to forecast the future for the company's activities. Since the cost minimization directly relates to the company's profitability of which is representing operation efficiency that can be expressed as a fraction, respective transportation modeling methods can be solved using linear programming. The models were studied based on a real-life data and as example of transportation flow of containers of SMT transport and services Ltd, operating on Russian market was taken. Since the forecast of future activities can be also related to the company's strategic planning. The forecasting problem was solved by one of the most common forecasting techniques used in business life, namely the trend adjusted forecast approach.

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ABBREVIATIONS

VAM – Vogel’s Approximation Method

MODI – Modified Distribution Method

TORA – Windows-based software designed for operations research

MAD – Mean absolute deviation

MSE – Mean squared error

MAPE – Mean absolute percent error

RSFE – Running sum of forecast error

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Abstract

This research work represents transportation modeling approaches and forecasting techniques addressing the transportation flow of cargo containers with semi-processed goods on the selected routes from a certain number of suppliers with various production capacities to the certain points of destination. The aim is to achieve the minimum cost of transportation flow and to forecast the future for the company's activities. Since the cost minimization directly relates to the company's profitability of which is representing operation efficiency that can be expressed as a fraction, respective transportation modeling methods can be solved using linear programming. The models were studied based on a real-life data and as example of transportation flow of containers of SMT transport and services Ltd, operating on Russian market was taken. Since the forecast of future activities can be also related to the company's strategic planning. The forecasting problem was solved by one of the most common forecasting techniques used in business life, namely the trend adjusted forecast approach.

1 INTRODUCTION

1.1 Research question and secondary research

This research work was conducted based on real transportation activities of a company operating in Russia in order to analyze its approach to the transportation flow and based on theory, to develop the possibility of using transportation modeling technique in the company's activities. Also, based on real figures and quantities, to find possible ways of solving problems of transportation activities in order to improve the transportation flow and to minimize the costs of shipping products from a series of sources to a series of destinations. SMT Transport and Services Ltd is operating in field of logistics and transportation, especially in organizing booking processes, loading and unloading of containers. The company itself is a part of transportation chain of ready-made goods and raw materials from Baltic countries, Finland and Russia to North Africa countries. The company also provides consultation services for a smaller producers and suppliers, in order to help them to achieve the required quality standards of their goods, and the selling in the international markets.

Cost minimization has become as one of important issues in business activities which have achieved a high priority especially today, when the economic slowdown has hit most of the business and production sectors. Cost rationalization has become an imperative for many companies to survive. Transportation modeling is one of those techniques that can help to find an optimum solution and save the costs in transportation activities. However, to achieve this goal by integrating or applying any of those methods and techniques to already existing system, the company's management can meet other problems and obstacles, where all parts of the transportation chain are equally important for the transportation flow processes.

The author of the thesis considers the study to be important, as the ability of minimizing transportation costs may affect in transportation planning process and long-term strategy for future operations and company profit potential. The main essential question of this research work is how transportation modeling may help to improve the transportation flow and minimize the costs of transportation. The answere to this main question involves minimizing the cost of shipping products from a series of sources to a

series of destinations. The main goal is to present three different methods of saving costs in transportation flow, showing the possibility of cost minmization by using these transportation modeling methods.
The methods were applied to the company‘s transportation planning activities, based on its existing quantities, by investigating the local suppliers in Russian territory. For the purpose of determining optimum solution in this particular case, the author investigated results of different transportation modeling methods, using hand calculations and TORA software to compare differences in the final results of each method and results from TORA software .

Several different transportation methods are used including both balanced and unbalanced cases. In the theoretical part, data collecting and presenting along with some solved examples are presented. The author used the Transportation Modeling methods such as Northwest-Corner rule, the Intuitive Lowest-Cost Method and Stepping-Stone method to compute the total cost of transportation, find an initial basic solution to the transportation problem and finally find the optimum solution.
The author investigated differences between results of transportation costs, by applying three different methods of transportation modeling. This approach helps to see the difference in results and therefore to develop a possibility of using transportation modeling methods in the future company activities.

Second research question is forecasting of the company's activity for the next year, based on existing data and figures of present activities. In the theoretical part of the forecast chapter, different methods introduced different types, methods and approaches of forecasting which may help to predict the future operations of the company and its position on the market. The author of the thesis considers the research work is of great importance as it provides very precise prediction and forecasts of the future values such as budget, future costs and profit of a company.

1.2 Background of the study

The company SMT Transport and Services Ltd is an international forwarding company operating in field of logistics and transportation, especially in organization of booking, loading and unloading containers for transporting of semi-processed goods. The company itself is a part of transportation chain of raw materials and semi-processed goods from Baltic countries, Finland, Russia to the North Africa and Arabic markets. The major part of the company's activities in Russia corresponds to the timber industry. Russia is one of the biggest suppliers of the timber, raw materials and semi-processed goods out of wood around the world. SMT Transport and services Ltd cooperates directly with Finnish company RETS Timber OY Ltd and represents its interest in the areas of buying and transporting of goods in wood industry within the Russian territory. RETS Timber is a partly owned trading company by Stora Enso Timber (50%) and United sawmills (50%). RETS Timber is the market leader with a one-third share of the total market area. The company sells Nordic and Baltic products from Stora Enso Timber, United sawmills and other minor suppliers. All product sales to North Africa and Middle East countries are handled by RETS Timber in Finland and Stora Enso Timber Doo, Koper in Slovenia. These companies export soft wood products to market areas in Egypt, Saudi Arabia, Algeria, Lebanon, Tunisia and Morocco.

1.3 Purpose of the study and primary research

The thesis was conducted in order to introduce the transportation problem solutions by applying different methods of the transportation flow of a company, in order to find the points that could be improved and minimize transportation costs of the company. The thesis was also conducted in order to show how basic figures of transportation flow can be transfered into a transportation matrix which is the basis of any transportation problem. Understanding of transportation problem methods can help to find an optimum solution for the transportation flow. Based on calculations and results of different methods and approaches to the same transportation problem, using different cases when demand was and wasn't equal to supply were also investigated. The author was also looking into the forecasting problem to show how forecasting approaches can help to predict transportation activities of the company in the future.

The thesis studied, with the help of transportation modeling methods such as Northwest-corner, Lowest-Cost and Vogel's Approximation , using real figures and data such as location of the sawmills in Russia, destinations to the terminal, terminal expenses and freight cost of transportation from the terminal in Russia to the final destination. The study investigates possible ways of minimizing the cost of transportation by using handmade calculation and additionally TORA Optimization System Windows-based software. These two tools are helped to understand the details of the transportation algorithm by describing all steps involved.

The Sales Manager of SMT transport and services Ltd was provided the author with all needed information about the difference of the goods quality from different producers and all the data and figures needed for the case study.

2 THEORY

2.1 Transportation problem

Throughout last years the changing nature of logistics and supply-chain directed companies towards global operations, has had an obvious impact on the relative importance of the different modes of transportation. In a global context, more production facilities are moved for greater distances because companies have developed the concept of focus factories, with a single global manufacturing point for certain products, and the concentration of production facilities in low-cost manufacturing locations.

Transportation problem became one of the most actual tasks for many companies. In any business activities, locations of the new production facilities, warehouses and distribution centers are the strategic issue with substantial cost implications where most companies usually consider and evaluate several locations. There are a wide variety of objective and subjective factors that must be always considered in finding the most rational decision. Depending on the sort of activity, for different companies and business industries the transportation problem can be solved using different methods, approaches and techniques. One of those methods is transportation modeling which is very common approach in solving transportation problem where solution considers alternative location within the framework of an existing distribution system.

2.2 The Transportation Model and its Variants

"The transportation model is a special class of linear programs that deals with shipping a commodity from sources (e.g., factories) to destinations (e.g., warehouses). The objective of the model is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. The model assumes that the shipping cost is proportional to the number of units shipped on a given route. In general, the transportation model can be extended to other areas of operation, including, among others, inventory control, employment scheduling, and personnel assignment"[1].

"The general problem of the ransportation model can be defined and represented by the network in Figure 1. There are *m* sources and *n* destinations, each represented by a node. The arcs represent the routes linking the sources and destinations. Arc (*i,j*) joining source *i* to destination *j* carries two pieces of information: the transportation cost per unit, C_{ij}, and the amount shipped, X_{ij}. The amount of supply at source *i* is a_i, and the amount of demand at destionation *j* is b_j. The objective of the model is to determine the unknows X_{ij} that will minimize the total transportation cost while satisfying the supply and demand restrictions"[2].

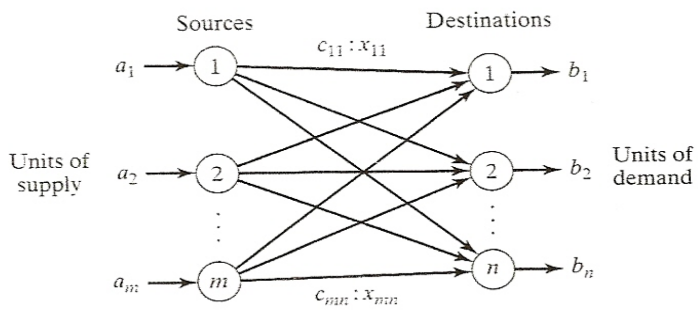

Figure 1. Representation of the transportation model with nodes and arcs

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[1] Hamdy A Taha, Prentice Hall 2002. *Operations Research: An introduction 7th Edition, p.165*

[2] Hamdy A Taha, Prentice Hall 2002. *Operations Research: An introduction 7th Edition, p.165*

2.3 Transportation matrix

According to Jay Heizer and Barry Render, "Transportation modeling is an iterative procedure for solving problems that involve minimizing the cost of shipping products from a series of sources to a series of destinations"[3]. Transportation modeling finds the least- cost means of shipping supplies from several origins to several destinations. Origin points of sources can be factories, warehouses, car rental agencies, like Avis, Hertz or any other points from which goods are shipped. Destinations are any points that receive goods. To use the transportation model, the following information must be concidered:

1. The origin points and the capacity or supply per period at each.
2. The destination points and the demand per period at each.
3. The cost of shipping one unit from each origin to each destination.

The way of how to built and transfer data from a real case into transportation matrix represented in the following pictures and case example of the Arizona plumbing company which makes, among other products, a full line of bathtubs. In this case firm must decide which of its factories should supply which of its warehouses.
Collecting data of the transportation problem:

From \ To	Albuquerque	Boston	Cleveland
Des Moines	$5	$4	$3
Evansville	$8	$4	$3
Fort Lauderdale	$9	$7	$5

Table 1. Transportation Costs per bathtube for Arizona Plumbing

The Table 1 represents the set of data for Arizona Plumbing, such as shipping costs of one bathtube from its factories to its warehauses. For example, the shipping cost of one bathtube for Arizona Plumbing from its factory in Des Moines to its Albuquerque warehause is 5$, 4$ to Boston and 3$ to Cleveland.

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[3] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.688*

Figure 2 shows that the 300 units required by Arizona Plumbing's Albuquerque warehouse might be shipped in various combinations from its Des Moines, Evansville, and Fort Lauderdale factories.

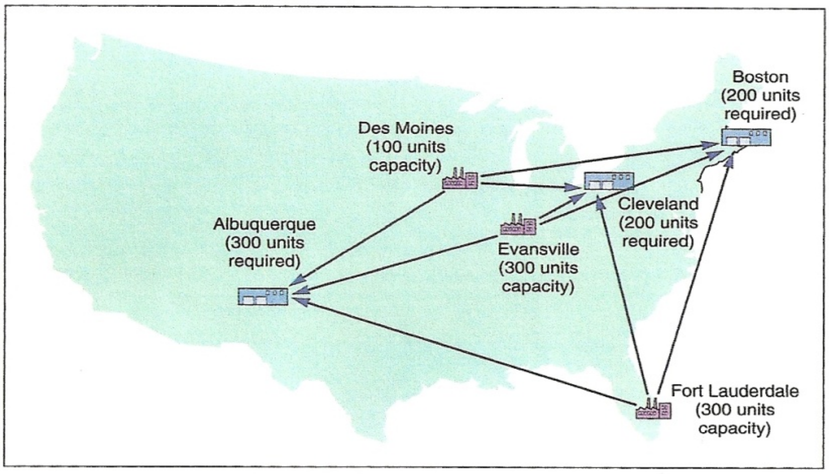

Figure 2. Scheme of transportation problem

"The first step in the modeling process is to set up a transportation matrix. Its purpose is to summarize all relevant data and to keep track of algorithm computations"[4].

Table 2 represents how transportation matrix can be constructed, based on the information displayed in Table 1 and Figure 2.

From \ To	Albuquerque	Boston	Cleveland	Factory capacity
Des Moines	$5	$4	$3	100
Evansville	$8	$4	$3	300
Fort Lauderdale	$9	$7	$5	300
Warehouse requirement	300	200	200	700

Des Moines capacity constraint

Cell representing a possible source-to-destination shipping assignment (Evansville to Cleveland)

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total demand and total supply

Table 2. Transportation matrix for Arizona Plumbing

[4] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.689*

2.4 Transportation modeling methods

Based on theory, "after all needed data was arranged in tabular form, the nex step of the technique is to establish an initial feasible solution to the problem"[5].

With the reference to the transportation problem the following terms are to be defined:

1. Feasible Solution, which is a set of non-negative allocations $X_{ij} \geq 0$ which satisfies the row and column restrictions.
2. Basic Feasible Solution, which is a feasible solution to a *m* - origin and *n*-destination problem if the number of positive allocations are (m+n–1). If the number of allocations in a basic feasible solutions are less than (m+n–1), it is called degenerate basic feasible solution (otherwise non-degenerate).
3. Optimal Solution is a feasible solution (not necessarily basic) if it minimizes the total transportation cost.

There are three different methods to obtain the initial basic solution of a transportation problem. These are Northwest-Corner Rule, Lowest cost entry and Vogel's approximation methods.

2.4.1 The Northwest – Corner Rule

"The Northwest-Corner Rule is a procedure in the transportation model where one starts at the upper left-hand cell of a table (the northwest corner) and systematically allocates units to shipping routes"[6].

Based on theory and using data from the previous transportation matrix of Arizona Plumbing the Northwest-Corner Rule can be represented as following:

1. Exhaust the supply (factory capacity) of each row (e.g., Des Moines:100) before moving down to the next row.
2. Exhaust the (warehouse) requirement of each column (e.g., Albuquergue: 300) before moving to the next column on the right.

[5] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.689*

[6] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.690*

3. Check to ensure that all suppliers and demands are met.

Table 3 shows the way of how to find an initial feasible solution to the Arizona Plumbing problem. The problem has been solved using the following steps:

1. Assign 100 tubs from Des Moines to Albuquergue (exhausting Des Moine's supply)
2. Assign 200 tubs from Evansville to Albuquergue (exhausting Albuquerque's demand)
3. Assign 100 tubs from Evansville to Boston (exhausting Evansville's supply)
4. Assign 100 tubs from Fort Lauderdale to Boston (exhausting Boston's demand)
5. Assign 200 tubs from Fort Lauderdale to Cleveland (exhausting Clevelan's demand and Fort Lauderdale's supply)

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5 100	\$4	\$3	100
(E) Evansville	\$8 200	\$4 100	\$3	300
(F) Fort Lauderdale	\$9	\$7 (100)	\$5 200	300
Warehouse requirement	300	200	200	700

Means that the firm is shipping 100 bathtubs from Fort Lauderdale to Boston

Table 3.Northwest-Corner solution to Arizona Plumbing Problem

The last step of each method is computing the total cost of shipping assignment. The total cost of Arizona Plumbing assignment represented in the Table 4.

Route				
From	To	Tubs Shipped	Cost per Unit	Total Cost
D	A	100	\$5	\$ 500
E	A	200	8	1,600
E	B	100	4	400
F	B	100	7	700
F	C	200	5	\$1,000
				Total: \$4,200

Table 4. Computed shipping cost for Arizona Plumbing

“The solution given is feasible because it satisfies all demand and supply contrains. Northwest Corner Rule is easy to use, but this method is totally ignores the costs ”[7].

2.4.2 The Intuitive Lowest- Cost Method

“The Intuitive Lowest- Cost Method is a cost- based approach in an initial solution to a transportation problem. This method makes initial allocations based on lowest cost ”[8]. Table 5 shows the way of how to find an initial feasible solution to the Arizona Plumbing problem, using Intuitive Lowest – Cost Method. This straightforward approach uses the following steps:

1. Identify the cell with the lowest cost. Break any ties for the lowest cost arbitrarily.
2. Allocate as many units as possible to that cell without exceeding the supply or demand.
3. Then cross out that row or column (or both) that is exhausted by this assignment.
4. Find the cell with the lowest cost from the remaining (not crossed out) cells.
5. Repeat steps 2 and 3 until all units have been allocated.

The total cost of Lowest – Cost Method method and how all the the steps described above were applied to the Arizona Plumbing problem, represented in the Table 5.

The total cost of this approach is = $3(100) + $3(100) + $4(200) + $9(300) = $4,100.
(D to C) (E to C) (E to B) (F to A)

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5	$4	$3 100	100
(E) Evansville	$8	$4 200	$3 100	300
(F) Fort Lauderdale	$9 300	$7	$5	300
Warehouse requirement	300	200	200	700

First, cross out top row (D) after entering 100 units in $3 cell because row D is satisfied.
Second, cross out column C after entering 100 units in this $3 cell because column C is satisfied.
Third, cross out row E and column B after entering 200 units in this $4 cell because a total of 300 units satisfies row E.
Finally, enter 300 units in the only remaining cell to complete the allocations.

Table 5. Intuitive Lowest-Cost Solution for Arizona Plumbing Problem

[7] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.690*

[8] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.690*

The total cost with the intuitive lowest-cost method is $4100 which is less than result of the Northwest-Corner method of $4200. The result of the second approach is showing that assignment has been improved in minimizing the costs by $100. Based on theory, "The northwest-corner and the intuitive lowest-cost approaches are meant only to provide us with a starting point; we often will have to employ an additional procedure to reach an optimal solution"[9].

2.4.3 The Vogel's Apploximation Method

"Vogel's Approximation Method (VAM) is the other important technique in addition to the northwest- corner and intuitive lowest-cost method. VAM is not quite as simple as the northwest corner approach, but it facilitates a very good initial solution – as a matter of fact, one that is often the optimal solution. Vogel's approximation method tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative"[10]. The first step of VAM, is to compute for each row and column the penalty faced if company should ship over the second best route instead of the least-cost route.

The following tables and calculations will step by step represent all six steps involved an initial VAM solution for Arizona Plumbing.

Step 1: For each row and column of the transportation table, find the difference between the two lowest unit shipping costs. These numbers represent the difference between the distribution cost on the best route in the row or column and the second best route in the row or column. (This is the opportunity cost of not using the best route.)

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[9] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.691*

[10] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, CD Tutorial, T 4-4*

FROM \ TO	Warehouse at Albuquerque	Warehouse at Boston	Warehouse at Cleveland	Factory Capacity
Des Moines factory	$5	$4	$3	100
Evansville factory	$8	$4	$3	300
Fort Lauderdale factory	$9	$7	$5	300
Warehouse requirements	300	200	200	700

Des Moines capacity constraint

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

Cleveland warehouse demand

Total demand and total supply

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Table 6.Transportation table for Arizona Plumbing

Step 1 has been done in Table 6. The numbers at the heads of the columns and to the right of the rows represent these differences. For example, in row E the three transportation costs are $8, $4, and $3. The two lowest costs are $4 and $3; their difference is $1.

Step 2: The process of identification the row or column with the greatest opportunity cost, or difference. In the Table 7, the row or column selected is column *A*, with a difference of 3.

	3	0	0		
FROM \ TO	ALBUQUERQUE *A*	BOSTON *B*	CLEVELAND *C*	TOTAL AVAILABLE	
DES MOINES *D*	5	4	3	100	1
EVANSVILLE *E*	8	4	3	300	1
FORT LAUDERDALE *F*	9	7	5	300	2
TOTAL REQUIRED	300	200	200	700	

Table 7.Transportation table with VAM Row and Column Differences Shown

Step 3: Assign as many units as possible to the lowest cost square in the row or column selected. Step 3 has been done in Table 8. Under Column A, the lowest-cost route is D–A (with a cost of $5), and 100 units have been assigned to that square. No more were placed in the square because doing so would exceed D’s availability.

Step 4: Eliminate any row or column that has just been completely satisfied by the assignment just made. This can be done by placing Xs in each appropriate square. Step 4 has been done in Table 8, D row. No future assignments will be made to the D–B or D–C routes.

Step 5: Recalculate the cost differences for the transportation table, omitting rows or columns crossed out in the preceding step. This is also shown in Table T4.6. A's, B's, and C's differences each change. D's row is eliminated, and E's and F's differences remain the same as in Table 8.

	~~3~~ 1	~~0~~ 3	~~0~~ 2		
FROM \ TO	A	B	C	TOTAL AVAILABLE	
D	[5] 100	[4] X	[3] X	100	~~1~~
E	[8]	[4]	[3]	300	1
F	[9]	[7]	[5]	300	2
TOTAL REQUIRED	300	200	200	700	

Table 8.VAM Assignment with D's Requirements Satisfied

Step 6: Return to step 2 and repeat the steps until an initial feasible solution has been obtained.

	~~3~~ 1	~~0~~ 3	~~0~~ 2		
FROM \ TO	A	B	C	TOTAL AVAILABLE	
D	[5] 100	[4] X	[3] X	100	~~1~~
E	[8]	[4] 200	[3]	300	~~1~~ 5
F	[9]	[7] X	[5]	300	~~2~~ 4
TOTAL REQUIRED	300	200	200	700	

Table 9.VAM Assignment with B's Requirements Satisfied

FROM \ TO	A	B	C	TOTAL AVAILABLE
D	(5) 100	(4) X	(3) X	100
E	(8) X	(4) 200	(3) 100	300
F	(9)	(7) X	(5)	300
TOTAL REQUIRED	300	200	200	700

Table 10.VAM Assignment with C's Requirements Satisfied

In this case, column *B* now has the greatest difference, which is equal to 3. We assign 200 units to the lowest- cost square in column *B* that has not been crossed out. This is seen to be *E–B*. Since *B*'s requirements have now been met, we place an *X* in the *F–B* square to eliminate it. Differences are once again recomputed. This process is summarized in Table 9.

The greatest difference is now in row *E*. Hence, we shall assign as many units as possible to the lowest-cost square in row *E*, that is, *E–C* with a cost of $3. The maximum assignment of 100 units depletes the remaining availability at *E*. The square *E–A* may therefore be crossed out. This is illustrated in Table T10. The final two allocations, at *F–A* and *F–C*, may be made by inspecting supply restrictions (in the rows) and demand requirements (in the columns). We see that an assignment of 200 units to *F–A* and 100 units to *F–C* completes the table (see Table 11).

FROM \ TO	A	B	C	TOTAL AVAILABLE
D	(5) 100	(4) X	(3) X	100
E	(8) X	(4) 200	(3) 100	300
F	(9) 200	(7) X	(5) 100	300
TOTAL REQUIRED	300	200	200	700

Table 11.Final Assignments to Balance Column and Row requirements

The total cost of this VAM assignment is = (100 units x $5) + (200 units x $4) + (100 units x $3) + (200 units x $9) + (100 units x $5) = $3,900

It is worth noting that the use of Vogel's approximation method on the Arizona Plumbing Corporation data produces the optimal solution to this problem. Even though VAM takes many more calculations to find an initial solution than does the northwest corner rule, it almost always produces a much better initial solution. Hence, VAM tends to minimize the total number of computations needed to reach an optimal solution.

2.4.4 Special issues in Modeling, Demand not equal to Supply

A common situation in real-world problems is a case in which total demand is not equal to total supply. Based on theory, "This situation can be easily handled using so-called unbalanced problems with the solution procedures by introducing dummy sources or dummy destinations. If total supply is greater than total demand, we make demand exactly equal the surplus by creating a dummy destination. Conversely, if total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand. Because these units will not in fact be shipped, we assign cost coefficients of zero to each square on the dummy location. In each case, then, the cost is zero"[11].

Example and Table 12 below for Arizona Plumbing Company, demonstrates the use of a dummy destination.
Let's assume that Arizona Plumbing increases the production in its Des Moines factory to 250 bathtubs, thereby increasing supply over demand. To reformulate this unbalanced problem, we refer back to the data presented in Table 1 and present the new matrix in Figure 2. First, we use the northwest-corner rule to find the initial feasible solution. Then, once the problem is balanced, we can proceed to the solution in the normal way.

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[11] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.695*

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Dummy	Factory capacity
(D) Des Moines	$5 250	$4	$3	0	250
(E) Evansville	$8 50	$4 200	$3 50	0	300
(F) Fort Lauderdale	$9	$7	$5 150	0 150	300
Warehouse requirement	300	200	200	150	850

New Des Moines capacity

Table 12.Northwest-Corner rule with Dummy

The total cost of Northwest –Corner method assignment with dummy destination is = (250 units x $5) + (50 units x $8) + (200 units x $4) + (50 units x $3) + (150 units x $5) + (150 units x $0) = $3,350

"Dummy sources are artificial shipping source points created in the transportation method when total demand is greater than total supply in order to affect a supply equal to the excess of demand over supply"[12].

"Dummy destinations are artificial destination points created in the transportation method when the total supply is greater than the total demand; they serve to equalize the total demand and supply"[13].

2.4.5 Optimization, method of multipliers

There is also another way to solve transportation problem, which is similar to the MODI method. This method is called the method of multipliers and its details are given in the following example.

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[12] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.695*

[13] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, p.695*

	1	2	3	4	Supply
1	10 / **5**	2 / **10**	20	11	**15**
2	12	7 / **5**	9 / **15**	20 / **5**	**25**
3	4	14	16	18 / **10**	**10**
Demand	**5**	**15**	**15**	**15**	

Table 13. Basic solution, Northwest-corner method

The determination of the entering variable from among the current non-basic variables (those that are not part of the starting basic solution) is done by computing the non-basic coefficients in, using the method of multipliers. In this method, task is to associate the multipliers U_i and V_i with row i and column j of the transportation tableau. For each current basic variable X_{ij} these multipliers are represented and must satisfy the following equations:

$$U_i + V_i = C_{ij} \text{ for each basic } X_{ij}$$

To solve these equations, the method of multipliers calls for arbitrarily setting $Ui = 0$, and then solving for the remaining variables as shown in the Table 14.

Basic variable	(u, v) equation	Solution
x_{11}	$u_1 + v_1 = 10$	$u_1 = 0 \rightarrow v_1 = 10$
x_{12}	$u_1 + v_2 = 2$	$u_1 = 0 \rightarrow v_2 = 2$
x_{22}	$u_2 + v_2 = 7$	$v_2 = 2 \rightarrow u_2 = 5$
x_{23}	$u_2 + v_3 = 9$	$u_2 = 5 \rightarrow v_3 = 4$
x_{24}	$u_2 + v_4 = 20$	$u_2 = 5 \rightarrow v_4 = 15$
x_{34}	$u_3 + v_4 = 18$	$v_4 = 15 \rightarrow u_3 = 3$

Table 14. Basic variables calculation

Finally the results are $U_1 = 0$, $U_2 = 5$, $U_3 = 3$, $V_1 = 10$, $V_2 = 2$, $V_3 = 4$, $V_4 = 15$.

In the next, U_i and V_i used to evaluate the non-basic variables by computing $U_i + V_i - C_{ij}$ for each non-basic X_{ij}. The results of these evaluations are shown in the Table 15.

Nonbasic variable	$u_i + v_j - c_{ij}$
x_{13}	$u_1 + v_3 - c_{13} = 0 + 4 - 20 = -16$
x_{14}	$u_1 + v_4 - c_{14} = 0 + 15 - 11 = 4$
x_{21}	$u_2 + v_1 - c_{21} = 5 + 10 - 12 = 3$
x_{31}	$u_3 + v_1 - c_{31} = 3 + 10 - 4 = \mathbf{9}$
x_{32}	$u_3 + v_2 - c_{32} = 3 + 2 - 14 = -9$
x_{33}	$u_3 + v_3 - c_{33} = 3 + 4 - 16 = -9$

Table 15. Nonbasic variables calculation

The preceding information, together with the fact that $U_i + V_j - C_{ij} = 0$ for each basic X_{ij}, is actually equivalent to computing the *z-row* of the simplex tableau as the following summary shows.

Basic	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	↓ x_{31}	x_{32}	x_{33}	x_{34}
z	0	0	−16	4	3	0	0	0	9	−9	−9	0

Table 16. Basic and Non-basic variables summary

Because the transportation model seeks to minimize the cost, the entering variable is the one which is having the most positive coefficient in the *z-row*. From the Table 16, X_{31} is the entering variable. According to theory, the preceding computations are usually done directly on the transportation tableau as shown in the Table 17, meaning that it is not necessary to write the (*U, V*) equations explicitly and start computing by setting $U_1 = 0$. The next step is to compute the *V*-values of all the columns that have basic variables in row 1, namely, V_1 and V_2. Next, we compute U_2 based on the *(U, V)* -equation of basic X_{22}. Now, based on given U_2 can be compute V_3 and V_4.

Finally, determination of U_3 using the basic equation of X_3. Once all the *U's* and *V's* have been determined, the non-basic variables can be calculated by computing $U_i + V_j - C_{ij}$ for each non-basic X_{ij}. These evaluations are shown in the Table 17 in the boxed southeast corner of each cell.

	$v_1 = 10$	$v_2 = 2$	$v_3 = 4$	$v_4 = 15$	Supply
$u_1 \equiv 0$	10 **5**	2 **10**	20 −16	11 4	**15**
$u_2 = 5$	12 3	7 **5**	9 **15**	20 **5**	**25**
$u_3 = 3$	4 9	14 −9	16 −9	18 **10**	**10**
Demand	**5**	**15**	**15**	**15**	

Table 17. Basic and Nonbasic variables in transportation tableau

Based on theory, having determined X_{31} as the entering variable, determination of the leaving variable is necessary. It is important to remember that if X_{31} enters the solution to become basic, one of the current basic variables must leave as non-basic (at "0" level).

The selection of X_{31} as the entering variable means that now goods must be shipped through this route because it reduces the total shipping cost. What is the most that can be shipped through the new route?

Observe in Table 17 that if route (3, 1) ships θ (i.e., $X_{31} = \theta$), then the maximum value of θ is determined based on two conditions:

1. Supply limits and demand requirements remain satisfied
2. Shipments through all routes must be nonnegative

These two conditions determine the maximum value of θ and the leaving variable in the following manner:

- First, construct a closed loop that starts and ends at the entering variable cell (3, 1). The loop consists of connected horizontal and vertical segments only (no diagonals are allowed). Except for the entering variable cell, each corner of the closed loop must coincide with a basic variable. Table 18 shows the loop for X_{31}. Exactly one loop exists for a given entering variable.

	$v_1 = 10$	$v_2 = 2$	$v_3 = 4$	$v_4 = 15$	Supply
$u_1 \equiv 0$	10 **5 − Θ** −	2 **10 + Θ** +	20 −16	11 4	**15**
$u_2 = 5$	12 3	7 **5 − Θ** −	9 **15**	20 **5 + Θ** +	**25**
$u_3 = 3$	4 **Θ** + 9	14 −9	16 −9	18 **10 − Θ** −	**10**
Demand	**5**	**15**	**15**	**15**	

Table 18. The loop for X_{31}

- Next step is to assign the amount of θ to the entering variable cell (3, 1). For the supply and demand limits to remain satisfied we must alternate between subtracting and adding the amount of θ at the successive corners of the loop as shown in Table 18 (it is immaterial if the loop is traced in a clockwise or counterclockwise direction). The new values of the variables then remain nonnegative if

$$X_{11} = 5 - \theta \geq 0$$
$$X_{22} = 5 - \theta \geq 0$$
$$X_{34} = 10 - \theta \geq 0$$

The maximum value of θ is 5, which occurs when both X_{11} and X_{22} reach “0” level. Because only one current basic variable must leave the basic solution, we can choose either X_{11} or X_{22} as the leaving variable. We arbitrarily choose X_{11} to leave the solution. The selection of X_{31} *(= 5)* as the entering variable and X_{11} as the leaving variable requires adjusting the values of the basic variables at the corners of the closed loop as Table 19 shows. The new cost is (15*$2)+(15*$9)+(10*$20)+(5*$4)+(5*$18) = $475.

	$v_1 = 1$	$v_2 = 2$	$v_3 = 4$	$v_4 = 15$	Supply
$u_1 = 0$	10 −9	2 **15 − Θ** (−)	20 −16	11 **Θ** (+) 4	**15**
$u_2 = 5$	12 −6	7 **0 + Θ** (+)	9 **15**	20 **10 − Θ** (−)	**25**
$u_3 = 3$	4 **5**	14 −9	16 −9	18 **5**	**10**
Demand	**5**	**15**	**15**	**15**	

Table 19. Adjusting the values of the basic variables at the corners of the closed loop

The computation of the multipliers u and v must be done again for the new basic solution, as Table 19 shows. The entering variable is X_{24}. The closed loop shows that X_{14} = 10 and that the leaving variable is x_{24}.The new solution, shown in the Table 20 with the total cost (5*$2) (10*$11) (10*$7) (15*$9) (5*$4) (5*$18) = $ 435which is less than the preceding one. Because the new $U_i + V_j – C_{ij}$ are now negative for all non-basic X_{ij}, the solution shown in Table 20 is optimal.

	$v_1 = -3$	$v_2 = 2$	$v_3 = 4$	$v_4 = 11$	Supply
$u_1 = 0$	10 −13	2 **5**	20 −16	11 **10**	**15**
$u_2 = 5$	12 −10	7 **10**	9 **15**	20 −4	**25**
$u_3 = 7$	4 **5**	14 −5	16 −5	18 **5**	**10**
Demand	**5**	**15**	**15**	**15**	

Table 20. The new and optimal solution

From silo	To mill	Number of truckloads
1	2	5
1	4	10
2	2	10
2	3	15
3	1	5
3	4	5
	Optimal cost = $ 435	

Table 21. Summarizing the optimum solution

2.5 TORA application

The TORA Optimization System is Windows-based software designed for use with many of the techniques represented in Operations management theory book. An important feature of the system is that it can be used to solve problems in a tutorial or automated mode. The tutorial mode is particularly useful because it allows concentrating on the main concepts of the algorithms while relieving you of the burden of the tedious computations that generally characterize Operations Research algorithms. TORA is totally self-contained, in the sense that all the instructions needed to drive the software are represented by menus, command buttons, check boxes, and the like. It requires no user manual.

2.6 Forecasting

2.6.1 The strategic importance of forecasting

Every day managers can make decisions without knowing what will happen in the future. They order inventory without knowing what sales will be, purchase new equipment despite uncertainty about demand for products, and make investments without knowing what the profits will be. Managers are always trying to make better estimations of what will happen in the future in the face of uncertainty. There are many different types of forecasts, forecasting models that managers can use to forecast and different methods of how to prepare, monitor, and judge the accuracy of a forecast. The main purpose of any forecast in business life is to make good estimates that will help forecaster to build the best strategy for the future activities. "Good forecasts are an essential part of efficient service and manufacturing operations"[14].

Forecasting is the art and science of predicting future events. It may involve taking historical data and projecting them into the future with some sort of mathematical model. It may be a subjective or intuitive prediction. Or it may involve a combination of

[14] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.104*

these-that is, a mathematical model: adjusted by a manager's good judgment. However, few businesses, can afford to avoid the process of forecasting by just waiting to see what happens and then taking their chances. Effective planning in both the short and long run depends on a forecast of demand for the company's products. “Good forecasts are of critical importance in all aspects of a business: The forecast is the only estimate of demand until actual demand becomes known. Forecasts of demand therefore drive decisions in many areas like human resource, capacity planning, and supply-chain management”[15] .

2.6.2 Types of forecasts

Type of forecasts:

1. Economic forecasts address the business cycle by predicting inflation rates, money supplies, housing starts, and other planning indicators.
2. Technological forecasts are concerned with rates of technological progress, which can result in the birth of exciting new products, requiring new plants and equipment.
3. Demand forecasts are projections of demand for a company's products or services. These forecasts, also called sales forecasts, drive a company's production, capacity, and scheduling systems and serve as inputs to financial, marketing, and personnel planning. Economic and technological forecasting are specialized techniques that may fall outside the role of the operations manager. The emphasis in this book will therefore be on demand forecasting.

A forecast is usually classified by the future time horizon that it covers. Time horizons fall into three categories:

1. Short-range forecast. This forecast has a time span of up to 1 year but is generally less than 3 months. It is used for planning purchasing, job scheduling, workforce levels, job assignments, and production levels.

2. Medium-range forecast. A medium-range, or intermediate, forecast generally spans from 3 months to 3 years. It is useful in sales planning, production planning and budgeting, cash budgeting, and analyzing various operating plans.

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[15] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.105*

3. Long-range forecast. Generally 3 years or more in time span, long-range forecasts are used in planning for new products, capital expenditures, facility location or expansion, and research and development.

Medium-range and long-range forecasts are distinguished from short-range forecasts by three features:

1. First, intermediate and long-run forecasts deal with more comprehensive issues and support management decisions regarding planning and products, plants, and processes.

2. Second, short-term forecasting usually employs different methodologies than longer-term forecasting. Mathematical techniques, such as moving averages, exponential smoothing, and trend extrapolation (all of which we shall examine shortly), are common to short-run projections.

Broader, less quantitative methods are useful in predicting such issue as whether a new product, like the optical disk recorder, should be introduced into a company's product line.

3. Finally, short-range forecasts tend to be more accurate than longer-range forecasts. Factors that influence demand change every day. Thus, as the time horizon lengthens, it is likely that one's forecast accuracy will diminish. It almost goes without saying, then, that sales forecasts must be updated regularly in order to maintain their value and integrity. "After each sales period, forecasts should be reviewed and revised "[16].

2.6.3 Forecasting approaches

"Based on theory, the forecasting follows seven basic steps which present a systematic way of initiating, designing, and implementing a forecasting system. When the system is to be used to generate forecasts regularly over time, data must be routinely collected. Then actual computations are usually made by computer"[17].

These steps are following:

1. Determine the use of the forecast.
2. Select the items to be forecasted.

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[16] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.105*

[17] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.106*

3. Determine the time horizon of the forecast. Is it short-, medium-, or long-term?
4. Select the forecasting model (s). The variety of statistical models such as moving averages, exponential smoothing, and regression analysis. It also employs judgmental, or non-quantitative, models.
5. Gather the data needed to make the forecast.
6. Make the forecast.
7. Validate and implement the results, considering forecast error.

"There are two general approaches to forecasting, just as there are two ways to tackle all decision modeling. One is quantitative analysis; the other is a qualitative approach. Quantitative forecasts use a variety of mathematical models that rely on historical data and/or causal variables to forecast demand. Subjective or qualitative forecasts incorporate such factors as the decision maker's intuition, emotions, personal experiences, and value system in reaching a forecast"[18]. Some firms use one approach and some use the other. In practice, a combination of the two is usually most effective.

"Forecasts are seldom perfect, which means that outside factors that cannot be predict or control often impact the forecast. Companies need to allow for this reality. Most forecasting techniques assume that there is some underlying stability in the system. Consequently, some firms automate their predictions using computerized forecasting software, and then closely monitor only the product items whose demand is erratic. Both product family and aggregated forecasts are more accurate than individual product forecasts."[19]

"In theory qualitative approach considers four different forecasting techniques such as jury of executive opinion, Delphi method, sales force composite, and consumer market survey."[20]

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[18] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.106*

[19] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.106*

[20] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.106*

Jury of executive opinion is the method, under which the opinions of a group of high-level experts or managers, often in combination with statistical models, are pooled to arrive at a group estimate of demand.

Delphi is the method where are three different types of participants: decision makers, staff personnel and respondents. Decision makers usually consist of a group of 5 to 10 experts who will be making the actual forecast. Staff personnel assist decision makers by preparing, distributing, collecting, and summarizing a series of questionnaire and survey results. The respondents are a group of people, often located in different places whose judgments are valued. This group provides inputs to the decision makers before the forecast is made.

Sales force composite is approach, where each salesperson estimates what sales will be in his or her region. These forecasts are then reviewed to ensure that they are realistic. Then they are combined at the district and national levels to reach an overall forecast. Consumer market survey is the method of solicits input from customers or potential customers regarding future purchasing plans. It can help not only in preparing a forecast but also in improving product design and planning for new products. The consumer market survey and sales force composite methods can, however, suffer from overly optimistic forecasts that arise from customer input.

"Quantitative approach consist five forecasting methods all of which use historical data and which can be divided into two groups, time-series models and associative model".[21] A time series is based on a sequence of evenly spaced (weekly, monthly, quarterly, and so on) data points. Forecasting time-series data implies that future values are predicted only from past values and those other variables, no matter how potentially valuable, may be ignored. Associative (or causal) models, such as trend progression and linear regression, incorporate the variables or factors that might influence the quantity being forecast. Analyzing time series means breaking down past data into components and

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[21] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.107*

then projecting them forward. A time series has four components: trend, seasonality, cycles, and random variation”[22].

Trend is the gradual upward or downward movement of the data over time. Changes in income, population, age distribution, or cultural views may account for movement in trend.
Seasonality is a data pattern that repeats itself after a period of days, weeks, months, or quarters.
Cycles are patterns in the data that occur every several years. They are usually tied into the business cycle and are of major importance in short-term business analysis and planning. Predicting business cycles is difficult because they may be affected by political events or by international turmoil.
Random variations are "blips" in the data caused by chance and unusual situations. They follow no discernible pattern, so they cannot be predicted.

Time-series models include naive approach, moving averages and exponential smoothing models. These models are based on prediction on the assumption that the future is a function of the past.

1. Naive approach – is a simplest way to forecast is to assume that demand in the next period will be equal to demand in the most recent period.
2. Moving averages - a forecasting method that uses an average of me most recent periods of data to forecast the next period. A moving-average forecast uses a number of historical actual data values to generate a forecast. Moving averages are useful if the forecaster can assume that market demands will stay fairly steady over time.
3. Exponential smoothing is a sophisticated weighted moving average forecasting technique in which data points are weighted by an exponential function. It involves very little record keeping of past data. The basic exponential smoothing can be represented as following:

[22] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.109*

New forecast = last period's forecast + α (last period's actual demand - last period's forecast),

Where, α is smoothing constant, chosen by forecaster, that has value between 0 and 1. According to Jay Heyzer and Barri Render, “the mathematical interpretation of this method can be shown as: $F_t = F_{t-1} + \alpha (A_{t-1} - F_{t-1})$,

where F_t - new forecast;

F_{t-1} - previous forecast;

α - smoothing constant (or weighting) constant ($0 \le \alpha \le 1$)

A_{t-1} – previous period’s actual demand

The smoothing constant, α is generally in the range from 0, 05 to 0, 50 for business applications. It can be changed to give more weight to recent data (when α is high) or more weight to past data (when α is low).”

“The exponential smoothing approach is easy to use and it has been successfully applied in virtually every type of business. However, the appropriate value of the smoothing constant, α can make the difference between an accurate forecast and an inaccurate forecast. High values of α, are chosen when the underlying average is likely to change. Low values of α, are used when the underlying average is fairly stable. In picking a value for the smoothing constant, the objective is to obtain the most accurate forecast.”[23]

4. Exponential smoothing with trend adjustment. Based on theory, exponential smoothing, the technique like any moving-average approach, fails to respond to trends. Exponential smoothing is a very popular approach in business. If a trend is a present, the exponential smoothing must be modified. The following example represents the way of how this approach can be modified.

The following table shows a severe lag in the 2nd, 3rd, 4th, and 5th months, even when our initial estimate for month 1 is perfect.

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[23] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.111*

MONTH	ACTUAL DEMAND	FORECAST FOR MONTH $T(F_T)$
1	100	$F_1 = 100$ (given)
2	200	$F_2 = F_1 + \alpha(A_1 - F_1) = 100 + .4(100 - 100) = 100$
3	300	$F_3 = F_2 + \alpha(A_2 - F_2) = 100 + .4(200 - 100) = 140$
4	400	$F_4 = F_3 + \alpha(A_3 - F_3) = 140 + .4(300 - 140) = 204$
5	500	$F_5 = F_4 + \alpha(A_4 - F_4) = 204 + .4(400 - 204) = 282$

Table 22.Severe lag in the 2nd, 3rd, 4th, 5th months

To improve the forecast, the more complex exponential smoothing model can be created, one that adjusts for trend. The idea is to compute an exponentially smoothed average of the data and then adjust for positive or negative lag in trend.

The new formula is: $(FIT_t) = (F_t) + (T_t)$,

where (FIT_t)- forecast including trend;

(F_t) - Exponentially smoothed forecast;

(T_t)- Exponentially smoothed trend;

With trend-adjusted exponential smoothing, estimates for both the average and the trend are smoothed. This procedure requires two smoothing constants, α for the average and β for the trend. The next step is to compute the average and trend each period:

$F_t = \alpha$ *(Actual demand last period) + (1 - α) (Forecast last period + Trend estimate last period)* or

$$F_t = \alpha (A_{t-1}) + (1-\alpha) (F_{t-1} + T_{t-1}) \qquad (1)$$

T_t = β (Forecast this period- Forecast last period) + (1-β) (Trend estimate last period), or

$$T_t = \beta(F_t - F_{t-1}) + (1-\beta) T_{t-1} \qquad (2)$$

Where Ft =exponentially smoothed forecast of the data series in period t

T_t=exponentially smoothed trend in period t

At =actual demand in period t

α =smoothing constant for the average ($0 \leq \alpha \leq 1$)

β =smoothing constant for the trend ($0 \leq \beta \leq 1$)

Based on theory, for computing forecast with trend adjustment the following steps should be done:

Step 1: Compute F_t, the exponentially smoothed forecast for period *t,* using equation (1)

Step 2: Compute the smoothed trend, T_t, using equation (2).

Step 3: Calculate the forecast including trend, FIT_t, by the formula $FIT_t = F_t + T_t$
The following example shows how to use trend-adjusted exponential smoothing.

As an example, all the steps of the forecast with trend adjustment are represented in Figure 3.

A large Portland manufacturer uses exponential smoothing to forecast demand for a piece of pollution-control equipment. It appears that an increasing trend is present.

Month (t)	Actual Demand (A_t)	Month (t)	Actual Demand (A_t)
1	12	6	21
2	17	7	31
3	20	8	28
4	19	9	36
5	24	10	?

Smoothing constants are assigned the values of $\alpha = .2$ and $\beta = .4$. Assume the initial forecast for month 1 (F_1) was 11 units and the trend over that period (T_1) was 2 units.

Step 1: Forecast for month 2:

$$F_2 = \alpha A_1 + (1-\alpha)(F_1 + T_1)$$
$$F_2 = (.2)(12) + (1 - .2)(11 + 2)$$
$$= 2.4 + (.8)(13) = 2.4 + 10.4 = 12.8 \text{ units}$$

Step 2: Compute the trend in period 2:

$$T_2 = \beta(F_2 - F_1) + (1-\beta)T_1$$
$$= .4(12.8 - 11) + (1 - .4)(2)$$
$$= (.4)(1.8) + (.6)(2) = .72 + 1.2 = 1.92$$

Step 3: Compute the forecast including trend (FIT_t):

$$FIT_2 = F_2 + T_2$$
$$= 12.8 + 1.92$$
$$= 14.72 \text{ units}$$

We will also do the same calculations for the third month.

Step 1: $F_3 = \alpha A_2 + (1-\alpha)(F_2 + T_2) = (.2)(17) + (1 - .2)(12.8 + 1.92)$
$= 3.4 + (.8)(14.72) = 3.4 + 11.78 = 15.18$

Step 2: $T_3 = \beta(F_3 - F_2) + (1-\beta)T_2 = (.4)(15.18 - 12.8) + (1 - .4)(1.92)$
$= (.4)(2.38) + (.6)(1.92) = .952 + 1.152 = 2.10$

Step 3: $FIT_3 = F_3 + T_3$
$= 15.18 + 2.10 = 17.28.$

Table 4.1 completes the forecasts for the 10-month period. Figure 4.3 compares actual demand to forecast including trend (FIT_t).

TABLE 4.1 Forecast with $\alpha = .2$ and $\beta = .4$

Month	Actual Demand	Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend FIT_t
1	12	11	2	13.00
2	17	12.80	1.92	14.72
3	20	15.18	2.10	17.28
4	19	17.82	2.32	20.14
5	24	19.91	2.23	22.14
6	21	22.51	2.38	24.89
7	31	24.11	2.07	26.18
8	28	27.14	2.45	29.59
9	36	29.28	2.32	31.60
10	—	32.48	2.68	35.16

Figure 3.Computing forecast with trend adjustment

The value of the trend-smoothing constant, β resembles α constant because a high β is more responsive to recent changes in trend. A low β gives less weight to the most recent trends and tends to smooth out the present trend. Values of β can be found by the trial-and-error approach or by using sophisticated commercial forecasting software, with the MAD used as a measure of comparison.

1. Trend projection - a time-series forecasting method that fits a trend line to a series of historical data points and then projects the line into the future for forecasts. This technique fits a trend line to a series of historical data points and then projects the line into the future for medium-to-long-range forecasts. Several mathematical trend equations can be developed (for example, exponential and quadratic).

2. Linear-regression analysis is the most common quantitative associative forecasting model, which is a straight-line mathematical model to describe the functional relationships between independent and dependent variables? The time-series associative forecasting models usually consider several variables that are related to the quantity being predicted. Once these related variables have been found, a statistical model is built and used to forecast the item of interest. This approach is more powerful than the time-series methods that use only the historic values for the forecasted variable. Many factors can be considered in an associative analysis.

2.6.4 Measuring Forecast error

The forecast error tells about how well the model performed against itself using pas data. The overall accuracy of any forecasting model-moving average, exponential smoothing or other- can be determined by comparing the forecasted values with the actual or observed values.

Based on theory, the forecast error or deviation of the period *t* can be defined as: *Forecast error = Actual demand - Forecast value =* $A_t - F_t$, where F_t denotes the forecast in period *t* and A_t denotes the actual demand in period *t.*

In theory, there are several measures commonly used in practice to calculate the overall forecast error. The measures can be used to compare different forecasting models, as well as to monitor forecasts to ensure they are performing well. Three of the most popular measures are mean absolute deviation (MAD), mean squared error (MSE), and mean absolute percent error (MAPE).
The Mean Absolute Deviation (MAD) is the first measure of the overall forecast error for a model. This value is computed by taking the sum of the absolute values of the individual forecast errors and dividing by the number of periods of data which represent the following formula:

$$MAD = \sum|actual - forecast|/n,$$ where *n* is the number of periods of data

The Mean Squared Error (MSE) is a second way of measuring overall forecast error. MSE is the average of the squared differences between the forecasted and observed values. Its formula is:

$$MSE = \sum (forecast\ errors)/n,$$ where n is number of periods of data.

Mean Absolute Percent Error is according to Jay Heizer and Barry Render "express the error as a percentage of the actual values"[24]. "A problem with both the MAD and MSE

[24] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.115*

is that their values depend on the magnitude of the item being forecast. If the forecast item is measured in thousand, the MAD and MSE values can be very large."[25] To avoid this problem, can be used the mean absolute percent error (MAPE), which is computed as the average of the absolute difference between the forecasted and actual values, expressed as a percentage of the actual values. That is, if there is a case with forecasted and actual values for n periods, the MAPE must be calculated using equation (3).

$$MAPE = \frac{100 \sum_{i=1}^{n} |actual\ i - forecast\ i| / actual\ i}{n} \quad (3)$$

where n is number of periods of data

2.6.5 Monitoring and controlling forecasts

Once a forecast has been completed, it should not be forgotten. No manager wants to be reminded that his or her forecast is horribly inaccurate, but a firm needs to determine why actual demand (or whatever variable is being examined) differed significantly from that projected.

Based on theory, one way to monitor forecasts to ensure that they are performing well is to use a tracking signal. A tracking signal is a measurement of how well the forecast is predicting actual values. As forecasts are updated every week, month, or quarter, the newly available demand data are compared to the forecast values. Positive tracking signals indicate that demand is greater than forecast. Negative signals mean that demand is less than forecast.

A good tracking signal-that is, one with a low RSFE-has about as much positive error as it has negative error. In other words, small deviations are okay, but positive and negative errors should balance one another so that the tracking signal centers closely around zero. The tracking signal is represented in equation 4 and computed as the running sum of the forecast errors (RSFE) divided by the mean absolute deviation (MAD).

$$(Tracking\ Signal) = \frac{RSFE}{MAD} = \frac{\sum(actual\ demand\ in\ period\ i - forecast\ demand\ in\ period\ i)}{MAD} \quad (4)$$

[25] Jay Heizer, Barry Render, Pearson Prentice Hall 2004. *Operations Management 7th Edition, Forecasting, p.114*

3 BASIC CALCULATIONS

3.1 The map and location of the main wood Suppliers

Figure 4. Wood suppliers location

Red Square **– Sawmill Pestovo**

Blue Square **– Sawmill Swedwood Karelia**

Green square **– Sawmill Swedwood Tihvin**

Yellow mark **- Final destination (terminals in Sankt Petersburg)**

3.2 Distances calculations from the biggest suppliers to the final destinations

3.2.1 Sawmill Pestovo

Pestovo Sawmill is one of the biggest Suppliers of sawn white wood and plywood in north-west region of Russia. The sawmill is located in the region of Novgorod and specialises in wood processing and procurement. ZAO Pestovo sawmill built by UPM-Kymmene in cooperation with the Russian timber company ZAO Novgorodlesprom. The Pestovo sawmill is a single-line circular sawmill and most of the production is exported, mainly to European and Asian markets. The business operations of this sawmill divided into two business areas: the sawmilling business and plywood business. The total yearly production capacity of sawmill is 260000m^3 sawn white wood and about 60000m^3 of planed products.

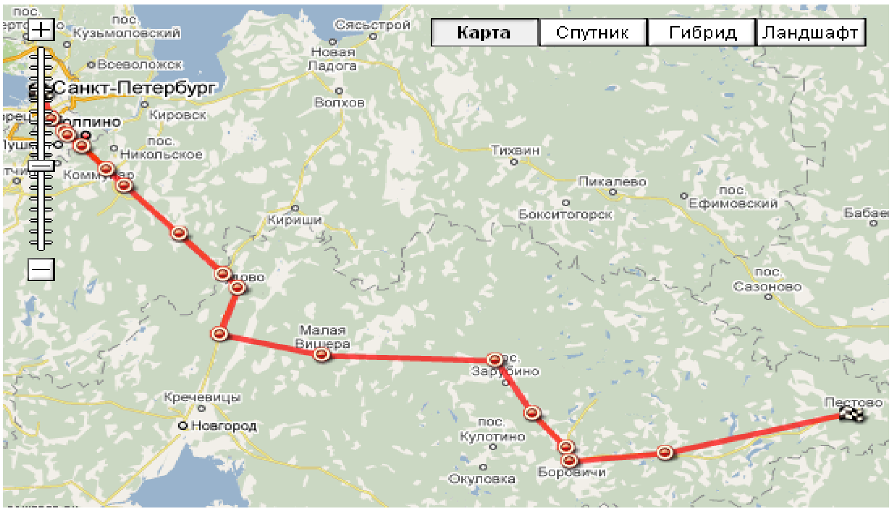

Figure 5. Distance calculation from sawmill Pestovo to the FCT terminal

The route from sawmill Pestovo to the First Container terminal, located in Sankt Petesburg is show in Figure 5. The total distance between sawmill Pestovo and First Container Terminal FCT (Sankt Petersburg) is 467 km.

Estimated time of transporting the goods by tracks to the terminal is 9, 5 hours.
Pre Carriage: Shipment costs 365 Euro/container.

3.2.2 Sawmill Swedwood Karelia, in Kostomuksha

Swedwood Karelia and swedwood Tihvin are the brunches of Swedwood Group which is a fully integrated international industrial group of IKEA.The main product which is company receiving from Swedwood sawmill in Kostomuksha is sawn white wood for Asian Markets. The total yearly production capacity of sawn white wood is 320000 m3

Figure 6. Distance calculation from Swedwood Karelia to the FCT terminal

The route from Swedwood Pestovo, located in Kostomuksha to the First Container terminal, located in Sankt Petesburg is show in Figure 6. The total distance between sawmill Swedwood Karelia and First Container Terminal FCT (Sankt Petersburg) is 919 km.

Estimated time of transporting the goods by tracks to the terminal is 16, 5 hours.

Pre Carriage: Shipment costs 625 Euro/container.

3.2.3 Sawmill Swedwood Karelia, in Tihvin

The total yearly production capacity of sawmill Tihvin is up to 500000m3

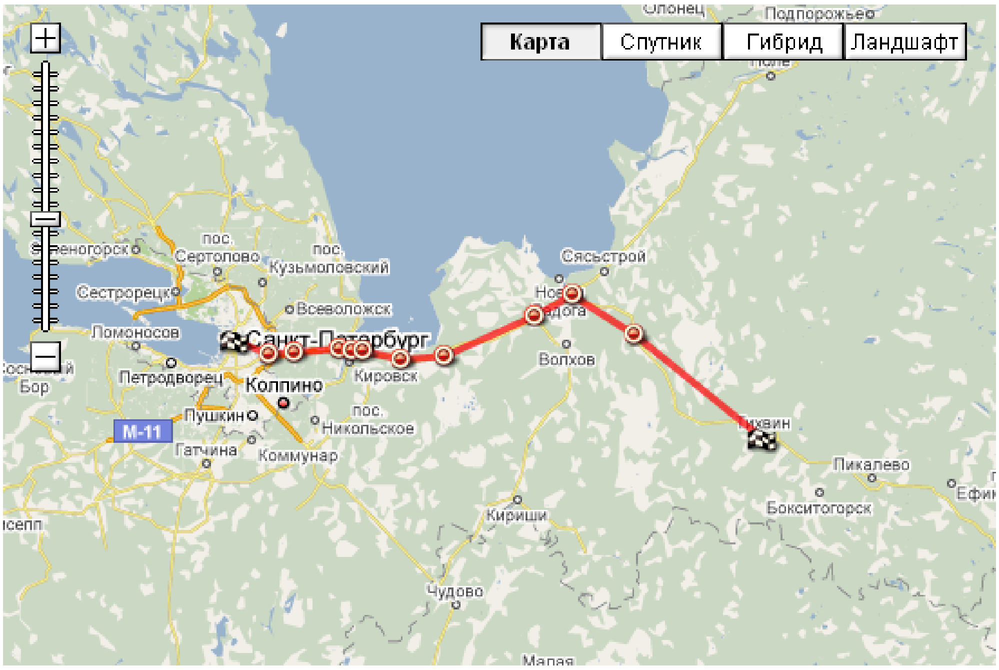

Figure 7. Distance calculation from Swedwood Tihvin sawmill to the FCT terminal

The route from sawmill Swedwood Tihvin to the First Container terminal, located in Sankt Petesburg is show in Figure 7. The total distance between sawmill Pestovo and First Container Terminal FCT (Sankt Petersburg) is 236 km.

Estimated time of transporting the goods by tracks to the terminal is 5 hours.

Pre Carriage: Shipment costs 330 Euro/container.

3.3 Freights, pricing and general scheme of the transportation flow

3.3.1 The scheme of transportation flow

Figure 8 shows transportation flow of the goods and relationship between suppliers and buyers through cargo terminals in Sankt Petersburg.

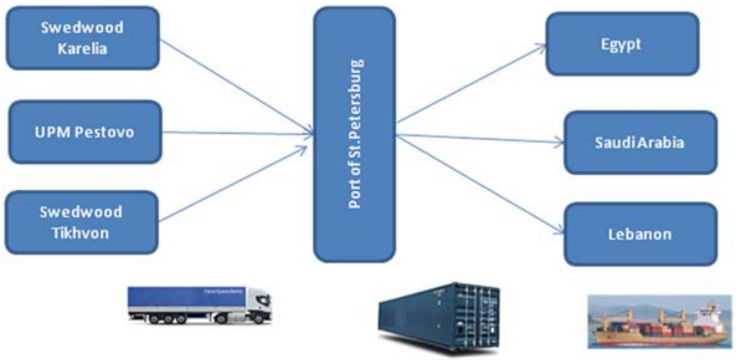

Figure 8. Transportatuion flow of the goods from Saw Mills to Arabic markets

From the Figure 6 the reader can see understand the process of transportation flow of the goods. The goods will be collected from sawmills by the trucks, delivered to the Sea port dry terminal where later on will be loaded into 40 m3 cargo containers. Last step is loading process to the ship and delivery of the goods to the final destination – Arabic market.

3.3.2 Pre Carriage Shipment costs

To From	Cargo terminal in St.Petersburg
Tihvin	330 Euro /Container
Pestovo	365Euro/ Container
Karelia	625 Euro/Container
Karelia	625 Euro/Container

Table 23. Pre Carriage Shipment costs

For the goods transportation from producers to the First Container Terminal in Sankt Petersburg, company is uses the services of the local transportation companies.

3.3.3 Loading and unloading expences

Table 24 represents the prices and charges of the loading and unloading processes. These prices are changeable according to the market condition and were given for the period of two months from 01.01.2010 to 01.03.2010 by the teminal cargo services department.

Unloading goods from the truck in the terminal	40 Euro /truck
Loading goods into container	220 Euro/Container
Containers transportation to the Sea Port territory for loading to the vessel	100 Euro/Container
Total costs of the loading and unloading services in the terminal	**360 Euro/Container**

Table 24. Description of terminal expences

3.3.4 Sea freights figures

Table 25 shows the sea freights per one container from the Seaport in Sankt Petersburg to the Seaports in Arabic countries.

From	**Final Destination**	**Price**
Sankt Petersburg seaport terminal	Alexandria Sea port, Egypt	**880 Euro/Cont.**
Sankt Petersburg seaport terminal	Jeddah Sea Port , Saudi Arabia	**745 Euro/Cont.**
Sankt Petersburg seaport terminal	Beirut Sea Port, Lebanon	**980 Euro/Cont.**

Table 25. Sea Freights figures

When needed quantity of the goods is ready to be transported from the wood suppliers to the Sea Port terminal, company starts the process of booking containers. The main shipping carriers and container holders that company use are Maersk, CMA, NYK and Evergreen. When goods arrive to the Sea terminal, their must be unloaded from the trucks and loaded into containers.

3.3.5 Calculations of total transportation costs from supplyer to the buyer

The total transportation costs is the sum of Pre Carrige Shipment costs, terminal services expences and Sea Freight costs. The total cost calculations from the biggest suppliers to the buyers are represented in the Table 26.

Buyer	Supplyer	Total transportation cost/ Cont. = Pre Carriage + Terminal Services expences + Sea Freight	Total price
Egypt	Pestovo Karelia Tihvin	365 + 360+ 880 625 + 360+ 880 330 + 360 + 880	**1605 Euro/Cont.** **1865 Euro/Cont** **1570 Euro/Cont**
Saudi Arabia	Pestovo Karelia Tihvin	365 + 360+ 745 625 + 360+ 745 330 + 360 + 745	**1470 Euro/Cont.** **1730 Euro/Cont** **1435 Euro/Cont**
Lebanon	Pestovo Karelia Tihvin	365 + 360+ 980 625 + 360+ 980 330 + 360 + 980	**1705 Euro/Cont.** **1965 Euro/Cont** **1670 Euro/Cont**

Table 26. Total transportation costs from the Sawmills to the final destination

4. TRANSPORTATION MODELING CALCULATIONS

Based on theory, such factors as total supply and total demand for each destination should be considered for building the transportation modeling matrix. Table 27 represents all needed data for the case study, such as see monthly demands of the products of company buyers and monthly supply of the company suppliers.

Supplier	Total capacity (year)	Monthly supply	Buyer	Monthly demand
Tihvin	Up to 1980 Containers	230 Containers	Egypt	265 Containers
Pestovo	Up to 2760 Containers	210 Containers	Saudi Arabia	160 Containers
Karelia	Up to 2520 Containers	165 Containers	Lebanon	105 Containers
Total	Up to 7260 Containers	605 Containers		530 Containers

Table 27. Demands and Quantities

4.1 Northwest - corner method (unbalanced)

The previous table shows that monthly supply of the goods is much higher than monthly demand of Asian market. Based on theory, in case when demand greater that supply, transportation problem call unbalanced and can be handled by the preceding solution procedures by introducing a dummy column, which is a slack variable, that will represent a fake warehouse requirement.

The Northwest- corner rule is easy to use, but this approach totally ignores the costs. This method requires to start from the upper left-hand cell (or northwest corner) of the transportation tableau and allocate units to shipping routes as shown in Table 28.

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply
Tihvin	1580€ 230	1435€	1670€	0€	230 Containers
Pestovo	1605€ 35	1510€ 160	1705€ 15	0€	210 Containers
Karelia	1865€	1730€	1965€ 90	0€ 75	165 Containers
Buyer Requi-rement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 28. Notrthwest-corner basic start solution

Route		Containers Shipped	Cost per Container	Total Cost
From	To			
Tihvin	Egypt	230	1580 €	363,400€
Pestovo	Egypt	35	1605 €	56,175 €
Pestovo	Saudi Arabia	160	1510 €	241,600 €
Pestovo	Lebanon	15	1705 €	25,575 €
Karelia	Lebanon	90	1965 €	176,850 €
Karelia	Dummy	75	0 €	0 €
Total Cost				863,600€

Table 29. Computing Shipping Cost Notrthwest-corner method

Total cost = 230x(1580 €) + 35x(1605 €) + 160x(1510 €) + 15x(1705 €) + 90x(1965 €) + 75x(0 €) = 363,400 € + 56,175 € + 241,600 € + 25,575 + 176,850 € + 0 = 863, 600 €

The total cost of this shipping assignment is 863,600.

Northwest –corner method optimization:

Based on theory, by using method of multipliers the first step of optimization processes is to find all basic and nonbasic variables of the basic solution.

1. Calculating *U* and *V* values, which marked as following: U_i – column of the transportation tableau , V_j – row of the transportation tableau.

For all basic variables (variables that are part of the solution) $U_i + V_j = C_{ij}$

Set $U_1 = 0;$

For X_{11} ; $U_1 + V_1 = C_{11};$ $U_1 = 0,\ V_1 = C_{11} - U_1 = 1580 - 0 = 1580;$

For X_{21} ; $U_2 + V_1 = C_{21};$ $V_1 = 1580,\ U_2 = C_{21} - V_1 = 1605 - 1580 = 25;$

For X_{22} ; $U_2 + V_2 = C_{22};$ $U_2 = 25,\ V_2 = C_{22} - U_2 = 1510 - 25 = 1485;$

For X_{23} ; $U_2 + V_3 = C_{23};$ $U_2 = 25,\ V_3 = C_{23} - U_2 = 1705 - 25 = 1680;$

For X_{33} ; $U_3 + V_3 = C_{33};$ $V_3 = 1680,\ U_3 = C_{33} - V_3 = 1965 - 1680 = 285;$

For X_{34} ; $U_3 + V_4 = C_{34};$ $U_3 = 285,\ V_4 = C_{34} - U_3 = 0 - 285 = -285;$

For all nonbasic variables (variables that are part of the solution), $U_i + V_j - C_{ij}$

Set $U_1 = 0;$

For $X_{12} = U_1 + V_2 - C_{12};$ $X_{12} = 0 + 1485 - 1435 = 50$; (the most positive)

For $X_{13} = U_1 + V_3 - C_{13};$ $X_{13} = 0 + 1680 - 1670 = 10;$

For $X_{14} = U_1 + V_4 - C_{14};$ $X_{14} = 0 +(- 285) - 0 = -285;$

For $X_{24} = U_2 + V_4 - C_{24};$ $X_{24} = 25 + (-285) - 0 = -260;$

For $X_{31} = U_3 + V_1 - C_{31};$ $X_{31} = 285 + 1580 - 1865 = 0;$

For $X_{32} = U_3 + V_2 - C_{34};$ $X_{32} = 285 + 1485 - 1730 = 40;$

Basic variables, pink color

	To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply
		V_1=1580	V_2 =1485	V_3 =1680	V_4 = -285	
U_1= 0	Tihvin	X_{11} 1580€ *230*	X_{12} 1435€ 50	X_{13} 1670€ 10	X_{14} 0€ -285	230 Containers
U_2= 25	Pestovo	X_{21} 1605€ *35*	X_{22} 1510€ *160*	X_{23} 1705€ *15*	X_{24} 0€ -260	210 Containers
U_3= 285	Karelia	X_{31} 1865€ 0	X_{32} 1730€ 40	X_{33} 1965€ *90*	X_{34} 0€ *75*	165 Containers
	Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 30. Basic and nonbasic variables

Nonbasic variables, blue color

Based on theory, the transportation model seeks to minimize the cost, the entering variable is the one having the most positive coefficient of nonbasic variables in any row. Since variable X_{12} has the most positive coefficient, for iteration 1 it will be the entering variable. Having determinated the entering variable, must be also determine the leaving variable, we need to determine the leaving variable. So, for instance if X_{12} enters the solution to become basic, one of the current basic variables must leave the solution as nonbasic at zero level. Table 31 shows the loop for X_{12}.

	To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply
		V_1=1580	V_2=1485	V_3=1680	V_4= -285	
U_1= 0	Tihvin	X_{11} 1580€ *230-θ* *(-)*	X_{12} 1435€ 50 *θ* *(+)*	X_{13} 1670€ 10	X_{14} 0€ -285	230 Containers
U_2= 25	Pestovo	X_{21} 1605€ *35 +θ* *(+)*	X_{22} 1510€ *160 – θ* *(-)*	X_{23} 1705€ *15*	X_{24} 0€ -260	210 Containers
U_3= 285	Karelia	X_{31} 1865€ 0	X_{32} 1730€ 40	X_{33} 1965€ *90*	X_{34} 0€ *75*	165 Containers
	Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 31. Optimization, Method of multipliers (Iteration 1)

Next step is to assign the amount of θ to the entering variable, in this case it's X_{12}. For the supply and demand limits to remain satisfied, must be find and alternative between substracting and adding the amount θ at the successive corners of the loop as shown in the Table 31. The new values of the variables then remain nonnegative if:

$$X_{11} = 230 - \theta \geq 0$$

$$X_{22} = 160 - \theta \geq 0$$

The maximum value of θ is 160, which occurs when X_{22} reach zero level, which will leave the solution and became the nonbasic variable.

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply
Tihvin	X_{11} 1580€ *70*	X_{12} 1435€ *160*	X_{13} 1670€	X_{14} 0€	230 Containers
Pestovo	X_{21} 1605€ *195*	X_{22} 1510€	X_{23} 1705€ *15*	X_{24} 0€	210 Containers
Karelia	X_{31} 1865€	X_{32} 1730€	X_{33} 1965€ *90*	X_{34} 0€ *75*	165 Containers
Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 32. New basic solution

Total cost = (70 *1580€) + (160*1435 €) + (195*1605 €) + (15*1705 €) + 90* (1965 €) + (75*0 €) = 110, 600€ + 229, 600€ + 312, 975€ + 25, 575 + 176, 850 € + 0 = 855, 600€

In the given new basic solution, all steps for computation of the multipliers *U* and *V* must be repeated again, until all nonbasic variables became negative. Calculating *U* and *V* values for the new basic solution shown in Table 32.

For all basic variables that are (variables that are part of the solution) $U_i + V_j = C_{ij}$ and

Set $U_1 = 0;$

For X_{11} ; $U_1 + V_1 = C_{11};$ $U_1 = 0,\ V_1 = C_{11} - U_1 = 1580 - 0 = 1580;$

For X_{12} ; $U_1 + V_2 = C_{12};$ $U_1 = 0,\ V_2 = C_{12} - U_1 = 1435 - 0 = 1435;$

For X_{21} ; $U_2 + V_1 = C_{21};$ $V_1 = 1580,\ U_2 = C_{21} - V_1 = 1605 - 1580 = 25;$

For X_{23} ; $U_2 + V_3 = C_{23};$ $U_2 = 25,\ V_3 = C_{23} - U_2 = 1705 - 25 = 1680;$

For X_{33} ; $U_3 + V_3 = C_{33};$ $V_3 = 1680,\ U_3 = C_{33} - V_3 = 1965 - 1680 = 285;$

For X_{34} *;* $U_3 + V_4 = C_{34};$ $\qquad$ $U_3 = 285,\ V_4 = C_{34} - U_3 = 0 - 285 = -285;$

For all nonbasic variables (that are not in the solution), $U_i + V_j - C_{ij}$ and *Set* $U_1 = 0;$

For X_{13} *;* $U_1 + V_3 - C_{13};$ $\qquad$ $X_{13} = 0 + 1680 - 1670 = 10;$ *(the most positive)*

For X_{14} *;* $U_1 + V_4 - C_{14};$ $\qquad$ $X_{14} = 0 + (-285) - 0 = -285;$

For X_{22} *;* $U_2 + V_2 - C_{22};$ $\qquad$ $X_{22} = 25 + 1435 - 1510 = -50;$

For X_{24} *;* $U_2 + V_4 - C_{24};$ $\qquad$ $X_{24} = 25 + (-285) - 0 = -260;$

For X_{31} *;* $U_3 + V_1 - C_{31};$ $\qquad$ $X_{31} = 285 + 1580 - 1865 = 0;$

For X_{32} *;* $U_3 + V_2 - C_{34};$ $\qquad$ $X_{32} = 285 + 1435 - 1730 = -10;$

Based on calculation of all nonbasic variables, the most positive number has the variable X_{13} which will be the entering variable for iteration 2.

		V_1=1580	V_2=1435	V_3=1680	V_4= -285	
	From \ To	**Egypt**	**Saudi Arabia**	**Lebanon**	**Dummy Buyer**	**Sawmills Capacity Supply**
U_1= 0	**Tihvin**	X_{11} 1580€ *70 - θ (-)*	X_{12} 1435€ *160*	X_{13} 1670€ *θ (+)* 10	X_{14} 0€ -285	230 Containers
U_2= 25	**Pestovo**	X_{21} 1605€ *195 +θ (+)*	X_{22} 1510€ -50	X_{23} 1705€ *15 – θ (-)*	X_{24} 0€ -260	210 Containers
U_3= 285	**Karelia**	X_{31} 1865€ 0	X_{32} 1730€ -10	X_{33} 1965€ *90*	X_{34} 0€ *75*	165 Containers
	Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 33. Optimization, Method of multipliers (Iteration 2)

$$X_{11} = 70 - \theta \geq 0$$

$$X_{23} = 15 - \theta \geq 0$$

The maximum value of θ is 15, which occurs when X_{23} reach zero level, which will leave the solution and became the nonbasic variable.

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply
Tihvin	X_{11} 1580€ 55	X_{12} 1435€ 160	X_{13} 1670€ 15	X_{14} 0€	230 Containers
Pestovo	X_{21} 1605€ 210	X_{22} 1510€	X_{23} 1705€	X_{24} 0€	210 Containers
Karelia	X_{31} 1865€	X_{32} 1730€	X_{33} 1965€ 90	X_{34} 0€ 75	165 Containers
Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 34. New basic solution

Total cost = (55*1580 €) + (160*1435 €) + (15*1670 €) + (210*1605 €) + (90*1965 €) + (75*0 €) = 86,900 € + 229,600€ + 25,050 + 337,050 € + 176,850 = 855,450 €

Calculating U and V values for the new basic solution shown in Table 34.

For all basic variables that are (that are part of the solution) $U_i + V_j = C_{ij}$ and $Set\ U_1 = 0;$

$For\ X_{11}\ ;\ U_1 + V_1 = C_{11};$ $\quad U_1 = 0,\ V_1 = C_{11} - U_1 = 1580 - 0 = 1580;$

$For\ X_{12}\ ;\ U_1 + V_2 = C_{12};$ $\quad U_1 = 0,\ V_2 = C_{12} - U_1 = 1435 - 0 = 1435;$

$For\ X_{13}\ ;\ U_1 + V_3 = C_{13};$ $\quad U_1 = 0,\ V_3 = C_{13} - U_1 = 1670 - 0 = 1670;$

$For\ X_{21}\ ;\ U_2 + V_1 = C_{21};$ $\quad V_1 = 1580,\ U_2 = C_{21} - V_1 = 1605 - 1580 = 25;$

For X_{33} *;* $U_3 + V_3 = C_{33}$*;* $V_3 = 1670,\ U_3 = C_{33} - V_3 = 1965 - 1670 = 295;$

For X_{34} *;* $U_3 + V_4 = C_{34}$*;* $U_3 = 295,\ V_4 = C_{34} - U_3 = 0 - 295 = -295;$

For all nonbasic variables (that are not in the solution), $U_i + V_j - C_{ij}$ and *Set* $U_1 = 0;$

For X_{14} *;* $U_1 + V_4 - C_{14}$*;* $X_{14} = 0 + (-295) - 0 = -295;$

For X_{22} *;* $U_2 + V_2 - C_{22}$*;* $X_{22} = 25 + 1435 - 1510 = -50;$

For X_{23} *;* $U_2 + V_3 - C_{23}$*;* $X_{23} = 25 + 1670 - 1705 = -10;$

For X_{24} *;* $U_2 + V_4 - C_{24}$*;* $X_{24} = 25 + (-295) - 0 = -270;$

For X_{31} *;* $U_3 + V_1 - C_{31}$*;* $X_{31} = 295 + 1580 - 1865 = 10;$ (the most positive)

For X_{32} *;* $U_3 + V_2 - C_{34}$*;* $X_{32} = 295 + 1435 - 1730 = 0;$

Based on calculation of all nonbasic variables above, the most positive number has the variable X_{31} which will be the entering variable for iteration 3.

	To / From	Egypt (V_1=1580)	Saudi Arabia (V_2=1435)	Lebanon (V_3=1670)	Dummy Buyer (V_4= -295)	Sawmills Capacity Supply
U_1= 0	Tihvin	X_{11} 1580€ *55-θ* *(-)*	X_{12} 1435€ *160*	X_{13} 1670€ *15+ θ* *(+)*	X_{14} 0€ -295	230 Containers
U_2= 25	Pestovo	X_{21} 1605€ *210*	X_{22} 1510€ -50	X_{23} 1705€ -10	X_{24} 0€ -270	210 Containers
U_3= 295	Karelia	X_{31} 1865€ *(+) θ* 10	X_{32} 1730€ 0	X_{33} 1965€ *90- θ*	X_{34} 0€ *75*	165 Cntainers
	Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 35. Optimization, Method of multipliers (Iteration 3)

$$X_{11} = 55 - \theta \geq 0$$

$$X_{23} = 90 - \theta \geq 0$$

The maximum value of θ is 55, which occurs when X_{11} reach zero level, which will leave the solution and became the nonbasic variable.

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply
Tihvin	X_{11} 1580€	X_{12} 1435€ *160*	X_{13} 1670€ *70*	X_{14} 0€	230 Containers
Pestovo	X_{21} 1605€ *210*	X_{22} 1510€	X_{23} 1705€	X_{24} 0€	210 Containers
Karelia	X_{31} 1865€ *55*	X_{32} 1730€	X_{33} 1965€ *35*	X_{34} 0€ *75*	165 Containers
Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 36. New basic solution

Total cost = (210*1605 €) + (55*1865 €) + (160*1435 €) + (70*1670 €) + (35*1965 €) + (75*0 €)= 337,050 € + 102, 575 € + 229,600 € + 116, 900 + 68, 775 € + 0 = 854, 900€

Calculating U and V values for the new basic solution shown Table 36.

For all basic variables that are (variables that are part of the solution) $U_i + V_j = C_{ij}$

Set $U_1 = 0;$

For X_{12} ; $U_1 + V_2 = C_{12};$ $U_1 = 0, V_2 = C_{12} - U_1 = 1435 - 0 = 1435;$

For X_{13} ; $U_1 + V_3 = C_{13};$ $U_1 = 0, V_3 = C_{13} - U_1 = 1670 - 0 = 1670;$

For X_{33} ; $U_3 + V_3 = C_{33};$ $V_3 = 1670, U_3 = C_{33} - V_3 = 1965 - 1670 = 295;$

For X_{34} ; $U_3 + V_4 = C_{34}$; $U_3 = 295$, $V_4 = C_{34} - U_3 = 0 - 295 = -295$;

For X_{31} ; $U_3 + V_1 = C_{31}$; $U_3 = 295$, $V_1 = C_{11} - U_3 = 1865 - 295 = 1570$;

For X_{21} ; $U_2 + V_1 = C_{21}$; $V_1 = 1570$, $U_2 = C_{21} - V_1 = 1605 - 1570 = 35$;

For all nonbasic variables (that are not in the solution), $U_i + V_j - C_{ij}$ and *Set* $U_1 = 0$;

For X_{11} ; $U_1 + V_1 - C_{11}$; $X_{11} = 0 + 1570 -1580 = - 10$;

For X_{14} ; $U_1 + V_4 - C_{14}$; $X_{14} = 0 + (-295) - 0 = -295$

For X_{22} ; $U_2 + V_2 - C_{22}$; $X_{22} = 35 + 1435 - 1510 = -40$;

For X_{23} ; $U_2 + V_3 - C_{23}$; $X_{23} = 35 + 1670 - 1705 = 0$;

For X_{24} ; $U_2 + V_4 - C_{24}$; $X_{24} = 35 + (-295) - 0 = -260$;

For X_{32} ; $U_3 + V_2 - C_{34}$; $X_{32} = 295 + 1435 - 1730 = 0$;

Since all nonbasic variables are negative or ”0” (which means no more most positive numbers) that means optimum solution which means minimum possible costs.

		V_1=1570	V_2=1435	V_3=1670	V_4= -295	
	To / From	**Egypt**	**Saudi Arabia**	**Lebanon**	**Dummy Buyer**	**Sawmills Capacity Supply**
U_1= 0	**Tihvin**	X_{11} 1580€ [-10]	X_{12} 1435€ *160*	X_{13} 1670€ *70*	X_{14} 0€ [-295]	230 Containers
U_2= 35	**Pestovo**	X_{21} 1605€ *210*	X_{22} 1510€ [-40]	X_{23} 1705€ [0]	X_{24} 0€ [-260]	210 Containers
U_3= 295	**Karelia**	X_{31} 1865€ *55*	X_{32} 1730€ [0]	X_{33} 1965€ *35*	X_{34} 0€ *75*	165 Containers
	Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 37. Optimization, Method of multipliers optimum solution (Iteration 4)

The total cost of optimum solution is (160 x 1435€) + (70 x 1670€) + (210 x 1605€) + (55 x 1865€) + (35 x 1965€) + (75 x 0€) = 229, 600€ + 116, 900€ + 337, 050€ + 102, 575€ + 68, 775€ + 0€ = 854, 900€ .

4.2 The Intuitive Lower-Cost method (unbalanced)

The Intuitive method is a cost-based approach in finding an initial solution to a transportation problem and makes initial allocations based on lower cost. This straightforward approach uses the following steps:

From \ To	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply
Tihvin	1580€ 70	1435€ 160	1670€	0€	230 Containers
Pestovo	1605€ 195	1510€	1705€ 15	0€	210 Containers
Karelia	1865€	1730€	1965€ 90	0€ 75	165 Containers
Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	75 Containers	605 Containers

Table 38. Intuitive Lower – Cost basic start solution

Route		Containers Shipped	Cost per Container	Total Cost
From	To			
Tihvin	Saudi Arabia	160	1435 €	229600 €
Tihvin	Egypt	70	1580 €	110600 €
Pestovo	Egypt	195	1605 €	312975 €
Pestovo	Lebanon	15	1705 €	25575 €
Karelia	Lebanon	90	1965 €	176850 €
Karelia	Dummy	75	0€	0 €
Total Cost				855,600 ,00€

Table 39. Computing Shipping Cost, Intuitive Lower – Cost method

Total cost = (160*1435 €) + (70*1580 €) + (195*1605 €) + (15*1705 €) + (90*1965€) + (75*0 €) = 229,600€ + 110,600€ + 312,975€ + 25,575 + 176,850 € +0 = 855, 600,00 €

The results of the Lower-Cost method is equal to the the final result of the total cost calculation of the previous Northwest-corner method. This is the case when the likelihood of a minimum cost solution does not improve the final result of intuitive method. These two methods are meant only to provide the reader with a starting point. To be able to reach the minimum cost and find an optimal solution, situation is required an additional procedure.

4.3 The Vogel Approximation method (unbalanced)

The VAM method is helping to move from an initial feasible solution to an optimal solution. This method is used to evaluate the cost effectiveness of shipping goods via transportation routes not currently in the solution as follows:

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply	
Tihvin	**1580€**	**1435€**	**1670€**	**0€**	**230Containers**	**1670**
Pestovo	**1605€**	**1510€**	**1705€**	**0€**	**210 Containers**	**1705**
Karelia	**1865€**	**1730€**	**1965€**	**0€** *75 Containers*	**~~165,~~ 90 Containers**	**1965**
Buyer Requirement total 530 Cont.	**265 Containers**	**160 Containers**	**105 Containers**	**~~75,~~ 0 Containers**	**605 Containers**	
	25	**75**	**35**			

Table 40. Vogel's Approximation Method, basic start solution 1st step

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply	
Tihvin	1580€	1435€ *160Containers*	1670€	0€	~~230~~,70 Containers	~~1670,~~ 145
Pestovo	1605€	1510€	1705€	0€	210 Containers	~~1705~~, 95
Karelia	1865€	1730€	1965€	0€	90 Containers	~~1965~~, 135
Buyer Requirement total 530 Cont.	265 Containers	~~160~~, 0 Containers	105 Containers	0 Containers	605 Containers	
	~~25~~, 25	~~75,~~ 75	~~35~~, 35			

Table 41. Vogel Approximation Method, basic start solution 2nd step

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply	
Tihvin	1580€	1435€	1670€	0€	70 Containers	~~1670, 145,~~ 90
Pestovo	1605€ *210Containers*	1510€	1705€	0€	~~210~~, 0 Containers	~~1705, 95,~~ 100
Karelia	1865€	1730€	1965€	0€	90 Containers	~~1965, 135,~~ 100
Buyer Requirement total 530 Cont.	~~265~~ , 55 Containers	0 Containers	105 Containers	0 Containers	605 Containers	
	~~25, 25,~~ 25	~~75, 75~~	~~35, 35,~~ 35			

Table 42. Vogel Approximation Method, basic start solution 3rd step

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply	
Tihvin	**1580€**	**1435€**	**1670€** *70 Containers*	**0€**	**~~70,~~ 0 Containers**	**~~1670, 145,~~ ~~90~~, 90**
Pestovo	**1605€**	**1510€**	**1705€**	**0€**	**0 Containers**	**~~1705, 95,~~ ~~100~~**
Karelia	**1865€**	**1730€**	**1965€**	**0€**	**90 Containers**	**~~1965, 135,~~ ~~100~~,10,0**
Buyer Requirement total 530 Cont.	**55 Containers**	**0 Containers**	**105 Containers**	**0 Containers**	**605 Containers**	
	~~25, 25, 25,~~ 285	**~~75~~**	**~~35, 35, 35,~~ 295**			

Table 43. Vogel Approximation Method, basic start solution 4th step

To / From	Egypt	Saudi Arabia	Lebanon	Dummy Buyer	Sawmills Capacity Supply	
Tihvin	**1580€**	**1435€** *160Containers*	**1670€** *70 Containers*	**0€**	**230 Containers**	**~~1670, 145, 90, 90~~**
Pestovo	**1605€** *210Containers*	**1510€**	**1705€**	**0€**	**210 Containers**	**~~1705, 95, 100~~**
Karelia	**1865€** *55 Containers*	**1730€**	**1965€** *35 Containers*	**0€** *75Containers*	**165 Containers**	**~~1965, 135, 100,100~~**
Buyer Requirement total 530 Cont.	**265 Containers**	**160 Containers**	**105 Containers**	**0 Containers**	**605 Containers**	
	~~25, 25, 25, 285~~	**~~75~~**	**~~35, 35, 35, 295~~**			

Table 44. Vogel Approximation Method, final distribution

Route		Containers Shipped	Cost per Container	Total Cost
From	To			
Tihvin	Saudi Arabia	160	1435€	229,600€
Tihvin	Lebanon	70	1670€	116,900€
Pestovo	Egypt	210	1605€	337,050€
Karelia	Egypt	55	1865€	102,575€
Karelia	Lebanon	35	1965€	68,775€
Karelia	Dummy	75	0€	0€
Total Cost				854,900€

Table 45. Vogel's Approximation Method, total cost calculation

Total cost calculation: (160 x 1435€) + (70 x1670€) + (210 x1605€) + (55 x 1865€) + (35 x 1965€) + (75 x 0€) = 229600 + 116900 + 337050 + 102575 + 68775 + 0 = 854,900€.

4.4 Northwest - corner method (balanced)

From \ To	Egypt	Saudi Arabia	Lebanon	Sawmills Capacity Supply
Tihvin	1580€ *230*	1435€	1670€	230 Containers
Pestovo	1605€ *35*	1510€ *160*	1705€ *15*	210 Containers
Karelia	1865€	1730€	1965€ *90*	165 Containers
Buyer Requirement total 530 Cont.	265 Containers	160 Containers	105 Containers	530 Containers

Table 46. Notrthwest-corner solution(balanced)

Route		Containers Shipped	Cost per Container	Total Cost
From	**To**			
Tihvin	**Egypt**	**230**	**1580 €**	**363,400€**
Pestovo	**Egypt**	**35**	**1605 €**	**56,175 €**
Pestovo	**Saudi Arabia**	**160**	**1510 €**	**241,600 €**
Pestovo	**Lebanon**	**15**	**1705 €**	**25,575 €**
Karelia	**Lebanon**	**90**	**1965 €**	**176,850 €**
Total Cost				**863,600 ,00€**

Table 47. Computing Shipping Cost Notrthwest-corner method

Total cost = (230*1580 €) + (35*1605 €) + (160*1510 €) + (15*1705 €) + (90*1965 €) = 363,400 € + 56,175 € + 241,600 € + 25,575 + 176,850 € = 863, 600,00 €

The total of this shipping assignment is 863,600 Euro.

Table 47 represents the Northwest-corner method in case when demand is equal to supply. This case can be taken into consideration and used for calculations, when ordering quantity and product supply are constant for a long period of time. Based on calculations and comparison analysis of the same method in both cases (Table 29 and Table 47), even thus when demand is equal and not equal to supply, the final results of the total costs of basic solution are the same.

4.5 TORA software calculations

The following tables are showing the results of TORA software solutions of the same transportation problem, where demand is not equal to supply. To be able to see and compare the final results of hand calculation and calculation using software, for TORA asignment author used the same data and quantities that has been used for hand calculations.

TORA Optimization System, Windows®-version 2.00

понедельник, Февраль 08, 2010 16:37

TRANSPORTATION MODEL-- ORIGINAL DATA

Title: Transportation problem Asian Market, Wood Supply

	Name	D1 Egypt	D2 Saudi	D3 Lebano	D4 Dummy	Supply
S1	Tihvin	1580,0000	1435,0000	1670,0000	0,0000	**230,0000**
S2	Pestov	1605,0000	1510,0000	1705,0000	0,0000	**210,0000**
S3	Kareli	1865,0000	1730,0000	1965,0000	0,0000	**165,0000**
Demand		**265,0000**	**160,0000**	**105,0000**	**75,0000**	

Figure 9. Original data of demand and supply

4.5.1 Northwest – corner method results:

TRANSPORTATION MODEL -- TABLEAUS (North-West Corner Method)

Title: Transportation problem Asian Market, Wood Supply

Iteration 1: **ObjVal 863600,0000**

	Name		D1 Egypt v1=1580,0000	D2 Saudi v2=1485,0000	D3 Lebano v3=1680,0000	D4 Dummy v4=-285,0000		**Supply**
S1	Tihvin	u1=0,0000	1580,0000 **230** 0,0000	1435,0000 50,0000	1670,0000 10,0000	0,0000 -285,0000	S1	**230**
S2	Pestov	u2=25,0000	1605,0000 **35** 0,0000	1510,0000 **160** 0,0000	1705,0000 **15** 0,0000	0,0000 -260,0000	S2	**210**
S3	Kareli	u3=285,0000	1865,0000 0,0000	1730,0000 40,0000	1965,0000 **90** 0,0000	0,0000 **75** 0,0000	S3	**165**
	Demand		**265**	**160**	**105**	**75**		

Figure 10. Northwest – corner, Iteration 1

Iteration 2: **ObjVal 855600,0000**

	Name		D1 Egypt v1=1580,0000	D2 Saudi v2=1435,0000	D3 Lebanon v3=1680,0000	D4 Dummy v4=-285,0000		**Supply**
S1	Tihvin	u1=0,0000	1580,0000 **70** 0,0000	1435,0000 **160** 0,0000	1670,0000 10,0000	0,0000 -285,0000	S1	**230**
S2	Pestov	u2=25,0000	1605,0000 **195** 0,0000	1510,0000 -50,0000	1705,0000 **15** 0,0000	0,0000 -260,0000	S2	**210**
S3	Kareli	u3=285,0000	1865,0000 0,0000	1730,0000 -10,0000	1965,0000 **90** 0,0000	0,0000 **75** 0,0000	S3	**165**
	Demand		**265**	**160**	**105**	**75**		

Figure 11. Northwest – corner, Iteration 2

Iteration 3: **ObjVal 855450,0000**

	Name		D1 Egypt v1=1580,0000	D2 Saudi v2=1435,0000	D3 Lebanon v3=1670,0000	D4 Dummy v4=-295,0000		**Supply**
S1	Tihvin	u1=0,0000	1580,0000 **55** 0,0000	1435,0000 **160** 0,0000	1670,0000 **15** 0,0000	0,0000 -295,0000	S1	**230**
S2	Pestov	u2=25,0000	1605,0000 **210** 0,0000	1510,0000 -50,0000	1705,0000 -10,0000	0,0000 -270,0000	S2	**210**
S3	Kareli	u3=295,0000	1865,0000 10,0000	1730,0000 0,0000	1965,0000 **90** 0,0000	0,0000 **75** 0,0000	S3	**165**
	Demand		**265**	**160**	**105**	**75**		

Figure 12. Northwest – corner, Iteration 3

Iteration 4: **ObjVal 854900,0000**

	Name		D1 Egypt v1=1570,0000	D2 Saudi v2=1435,0000	D3 Lebanon v3=1670,0000	D4 Dummy v4=-295,0000		**Supply**
S1	Tihvin	u1=0,0000	1580,0000 -10,0000	1435,0000 **160** 0,0000	1670,0000 **70** 0,0000	0,0000 -295,0000	S1	**230**
S2	Pestov	u2=35,0000	1605,0000 **210** 0,0000	1510,0000 -40,0000	1705,0000 0,0000	0,0000 -260,0000	S2	**210**
S3	Kareli	u3=295,0000	1865,0000 **55** 0,0000	1730,0000 0,0000	1965,0000 **35** 0,0000	0,0000 **75** 0,0000	S3	**165**
	Demand		**265**	**160**	**105**	**75**		

Figure 13. Northwest – corner, Iteration 4 (Optimum solution)

4.5.2 Least cost method

TRANSPORTATION MODEL -- TABLEAUS (Least-Cost Method)

Title: Transportation problem Asian Market, Wood Supply

Iteration 1: **ObjVal** **877225,000**

	Name		D1 Egypt v1=1530,000	D2 Saudi v2=1435,000	D3 Lebano v3=1630,000	D4 Dummy v4=0,000		Supply
S1	Tihvin	u1=0,000	1580,000 -50,000	1435,000 **155** 0,000	1670,000 -40,000	0,000 **75** 0,000	S1	**230**
S2	Pestov	u2=75,000	1605,000 **205** 0,000	1510,000 **5** 0,000	1705,000 0,000	0,000 75,000	S2	**210**
S3	Kareli	u3=335,000	1865,000 **60** 0,000	1730,000 40,000	1965,000 **105** 0,000	0,000 335,000	S3	**165**
	Demand		**265**	**160**	**105**	**75**		

Figure 14. Least cost method, Iteration 1

Iteration 2: **ObjVal** **875550,000**

	Name		D1 Egypt v1=1865,000	D2 Saudi v2=1435,000	D3 Lebanon v3=1965,000	D4 Dummy v4=0,000		Supply
S1	Tihvin	u1=0,000	1580,000 285,000	1435,000 **160** 0,000	1670,000 295,000	0,000 **70** 0,000	S1	**230**
S2	Pestov	u2=-260,000	1605,000 **210** 0,000	1510,000 -335,000	1705,000 0,000	0,000 -260,000	S2	**210**
S3	Kareli	u3=0,000	1865,000 **55** 0,000	1730,000 -295,000	1965,000 **105** 0,000	0,000 **5** 0,000	S3	**165**
	Demand		**265**	**160**	**105**	**75**		

Figure 15. Least cost method, Iteration 2

Iteration 3: **ObjVal** **854900,000**

	Name		D1 Egypt v1=1570,000	D2 Saudi v2=1435,000	D3 Lebanon v3=1670,000	D4 Dummy v4=-295,000		Supply
S1	Tihvin	u1=0,000	1580,000 -10,000	1435,000 **160** 0,000	1670,000 **70** 0,000	0,000 -295,000	S1	**230**
S2	Pestov	u2=35,000	1605,000 **210** 0,000	1510,000 -40,000	1705,000 0,000	0,000 -260,000	S2	**210**
S3	Kareli	u3=295,000	1865,000 **55** 0,000	1730,000 0,000	1965,000 **35** 0,000	0,000 **75** 0,000	S3	**165**
	Demand		**265**	**160**	**105**	**75**		

Figure 16. Least cost method, Iteration 3 (Optimum solution)

4.5.3 Vogel's Approximation Method

TRANSPORTATION MODEL -- TABLEAUS (Vogel's Method)

Title: Transportation problem Asian Market, Wood Supply

Iteration 1: **ObjVal** **854900,000**

	Name		D1 Egypt v1=1570,000	D2 Saudi v2=1435,000	D3 Lebano v3=1670,000	D4 Dummy v4=-295,000		Supply
S1	Tihvin	u1=0,000	1580,000 -10,000	1435,000 **160** 0,000	1670,000 **70** 0,000	0,000 -295,000	S1	**230**
S2	Pestov	u2=35,000	1605,000 **210** 0,000	1510,000 -40,000	1705,000 0,000	0,000 -260,000	S2	**210**
S3	Kareli	u3=295,000	1865,000 **55** 0,000	1730,000 0,000	1965,000 **35** 0,000	0,000 **75** 0,000	S3	**165**
	Demand		**265**	**160**	**105**	**75**		

Figure 17. Vogel's Approximation Least Method, Iteration 1 (Optimum solution)

TORA, Transportation Model Output Summary

TRANSPORTATION MODEL OUTPUT SUMMARY

Title: Transportation problem Asian Market, Wood Supply
Final Iteration No.: 3
Objective Value (minimum cost) =854900,000

From	To	Amt Shipped	Obj Coeff	Obj Contrib
S1: Tihvin	D2: Saudi	160	1435,000	229600,000
S1: Tihvin	D3: Lebanon	70	1670,000	116900,000
S2: Pestovo	D1: Egypt	210	1605,000	337050,000
S3: Karelia	D1: Egypt	55	1865,000	102575,000
S3: Karelia	D3: Lebanon	35	1965,000	68775,000
S3: Karelia	D4: Dummy	75	0,000	0,000

Figure 18.Transportation model, minimum cost output summary

4.6 Comparison analysis of TORA software and hand calculations

Comparison analysis is conducted in order to prove that the final results of operations research TORA software are the same as the results of hand made calculation using transportation modeling methods. The hand made calculations were conducted in order to show and describe all steps for solving transportation problem which helps to understand the nature of all transportation modeling processes to achieve an optimum solution and minimize the cost. Based on final results of hand made calculations of each transportation modeling method and calculations using TORA software the author can tell with confidence that mangers can use TORA software for solving transportation problems, because it's much faster and less time consuming approach.

The results of this research work obtained using software "TORA" and hand calculations. The software is accurate and efficient because the optimum solution are identical, which shows the accuracy efficiency of the software "TORA".

5 FORECASTING CALCULATIONS.

3.4 Forecasting with trend adjustments

In this part of the paper work the author wanted to show the forecast calculations for the future company sales in year 2010. Based on theory, exponential smoothing forecast with trend adjustment is one of the most common methods for predicting future in business activities. For computation of exponential smoothing forecast with trend adjustment author use real data of goods sold since year 2005. Table 48 shows the company's data of total quantities sold per year which was calculated in the end of December of each year.

Years	Containers sold
2004	4410
2005	4560
2006	4615
2007	4725
2008	4680
2009	4630

Table 48. Containers sold per year

Based on theory , exponential smoothing with trend adjustment forecast requires two smoothing constants, α for the average and β for the trend. In the calculation part of this thesis work , author shows few calculations using different numbers for the smoothing constants, which are between "0" and "1". The main reason of using different values for the smoothing constants is to show how different combinations of smoothing constants can affect on mean absolute error and the final results of the forecast with trend adjustment. The first calculation which was presented and fully discribed in mathematical expressions and shows all the steps of forecast with trend adjustment method. The other calculations were done in Microsoft office Excel, using basic mathematical formulas and applications in order to show the difference of final results of the forecasts and forecast errors. The author decided to begin with the basic values of the smoothing constants where α = 0,1 and β = 0,1. Initial forecast for the year 2004 was 4200 units and trend over that period was 50 units.

Computing forecast with trend adjustment:

Step 1: Calculating F_t, the exponential smoothing forecast for years 2005-2010 using formula: $F_t = \alpha (A_{t-1}) + (1-\alpha) (F_{t-1} + T_{t-1})$.

$F_{1\ (given)}$ = 4500 (the initial forecast for the year 2004)

$F_{2\ (year\ 2005)}$ = (0, 1) * (4410) + (1- 0, 1) * (4500 + 50) = 441 + 4095 = 4536 units;

$F_{3\ (year\ 2006)}$ = (0, 1) * (4560) + (1- 0, 1) * (4536 + 48, 6) = 456 + 4126, 14 = 4582, 14 units;

$F_{4\ (year\ 2007)}$ = (0, 1) * (4615) + (1- 0, 1) * (4582, 14 + 48, 35) = 461,5 + 4167, 44 = 4628,94 units;

$F_{5\ (year\ 2008)}$ = (0, 1) * (4725) + (1- 0, 1) * (4628, 94 + 48, 2) = 472, 5 + 4209, 42 = 4681, 93 units;

$F_{6\ (year\ 2005)}$ = (0, 1) * (4680) + (1- 0, 1) * (4681, 93 + 48, 68) = 468 + 4257, 54 = 4725, 55 units;

$F_{7\ (year\ 2010)}$ = (0, 1) * (4630) + (1- 0, 1) * (4725, 55 + 48, 17) = 463 + 4296, 34 = 4759, 35 units;

Step 2: Computing the smoothed trends, using formula $T_t = \beta (F_t - F_{t-1}) + (1-\beta) T_{t-1}$:

$T_{1\ (given)}$ = 50 (trend over period 2004)

$T_{2\ (year\ 2005)}$ = (0, 1) * (4536 - 4500) + (1-0, 1) * 50 = 3, 6 + 45 = 48, 6;

$T_{3\ (year\ 2006)}$ = (0, 1) * (4582, 14 - 4536) + (1-0, 1) * 48, 6 = 4, 61 + 43, 74 = 48, 35;

$T_{4\ (year\ 2007)}$ = (0, 1) * (4628, 94 - 4582, 14) + (1-0, 1) * 48, 35 = 4, 68 + 43, 51 = 48, 2;

$T_{5\ (year\ 2008)}$ = (0, 1) * (4681, 93 - 4628, 94) + (1-0, 1) * 48, 2 = 5, 29 + 43, 39 = 48, 68;

$T_{6\ (year\ 2009)}$ = (0, 1) * (4725, 55 - 4681, 93) + (1-0, 1) * 48, 68 = 4, 36 + 43, 81 = 48, 17;

$T_{7\ (year\ 2010)}$ = (0, 1) * (4759, 35 - 4725, 55) + (1-0, 1) * 48, 17 = 3, 38 + 43, 35 = 46, 73;

Step 3: Computing forecasts including trend, using formula $(FIT_t) = (F_t) + (T_t)$.

$FIT_{2\ (year\ 2005)} = F_2 + T_2$ = 4536 + 48, 6 = 4584, 6 units;

FIT_3 (year 2006) = $F_3 + T_3$ = 4582, 14 + 48, 35 = 4630, 49 units;

FIT_4 (year 2007) = $F_4 + T_4$ = 4628, 94 + 48, 2 = 4677, 14 units;

FIT_5 (year 2008) = $F_5 + T_5$ = 4681, 93 + 48, 68 = 4730, 61 units;

FIT_6 (year 2009) = $F_6 + T_6$ = 4725, 55 + 48, 17 = 4773, 72 units;

FIT_7 (year 2010) = $F_7 + T_7$ = 4759, 35 + 46, 73 = 4806, 08 units;

Computing forecast mean absolute percent error (MAPE), using formula:

$$MAPE = \frac{100 \sum_{i=1}^{n} |\text{actual } i - forecast\ i| / actual\ i}{n} \quad :$$

Years	Actual quantity	Forecast with α = 0,1; β = 0,1	Absolute percent Error 100 x (|error|/Actual)
2005	4560	4584, 60	100 * (|24,6|/4560) = 0, 54%
2006	4615	4630, 49	100 * (|15, 49|/4615) = 0, 34%
2007	4725	4677, 14	100 * (|47, 86|/4725) = 1,01%
2008	4680	4730, 61	100 * (|50, 61|/4680) = 1, 08%
2009	4630	4773, 72	100 * (|143, 72|/4630) = 3, 1%
			Total MAPE = **6,07%**

Table 49. Computing forecast mean absolute error (MAPE) with α = 0, 1 and β = 0, 1

Based on results from the previous calculation, the reader can see that the use of smoothing constants α = 0, 1 and β = 0, 1 are not so good, because the results of the forecasts are far from the results of the actual sales. The forecast error is 6, 07% which is quite high. In the Table 49 the author wanted to represent the results of the same calculation which was done in Microsoft Excel, using same figures of the smoothing constants of the forecast, α = 0, 1 and β = 0, 1. The others calculations with different smoothing constants were done to find the most suitable α and β for the case study and smallest forecast error.

Forecast 2004 (given)	Trend 2004 (given)	α	β
4500	50	0,1	0,1

Given data

Years	Actual sales	Forecast
2004	4410	4500,00
2005	4560	4536,00
2006	4615	4582,14
2007	4725	4628,94
2008	4680	4681,93
2009	4630	4725,55
2010		4759,35

Computing exponential smoothing forecasts

Years	Trend
2004	50
2005	48,60
2006	48,35
2007	48,20
2008	48,68
2009	48,17
2010	46,73

Computing trend

Years	Forecasts(F)	Trend (T)	Forecast with trend adjustment
2005	4536,00	48,60	4584,60
2006	4582,14	48,35	4630,49
2007	4628,94	48,20	4677,14
2008	4681,93	48,68	4730,61
2009	4725,55	48,17	4773,72
2010	4759,35	46,73	4806,08

Computing exponential smoothing forecasts with trend adjustment

Years	Actual sales	Forecast α = 0,1; β = 0,1;	Mean absolute percent Error 100% * (|Error|/Actual demand)
2005	4560		0,54%
2006	4615		0,34%
2007	4725		1,01%
2008	4680		1,08%
2009	4630		3,10%
		Total Error	**6,07%**

Computing forecast mean absolute error (MAPE)

Table 50. Computing exponential smoothing forecast with trend adjustment, (α=0.1;β=0.1)

Forecast 2004 (given)	Trend 2004 (given)	α	β
4500	50	0,9	0,8

Given data

Years	Actual sales	Forecast
2004	4410	4500,00
2005	4560	4424,00
2006	4615	4541,32
2007	4725	4616,00
2008	4680	4721,75
2009	4630	4694,16
2010		4636,21

Computing exponential smoothing forecasts

Years	Trend
2004	50
2005	-50,8
2006	83,70
2007	76,48
2008	99,89
2009	-2,09
2010	-46,7

Computing trend

Years	Forecasts(F)	Trend (T)	Forecast with trend adjustment
2005	4424,00	-50,8	4373,20
2006	4541,32	83,70	4625,02
2007	4616,00	76,48	4692,49
2008	4721,75	99,89	4821,64
2009	4694,16	-2,09	4692,08
2010	4636,21	-46,7	4589,42

Computing exponential smoothing forecasts with trend adjustment

Years	Actual sales	Forecast α = 0,9; β = 0,8;	Mean absolute percent Error 100% * (|Error|/Actual demand)
2005	4560		4,1 %
2006	4615		0,22%
2007	4725		0,69%
2008	4680		3,03%
2009	4630		1,34%
		Total Error	**9,38%**

Computing forecast mean absolute error (MAPE)

Table 51. Computing exponential smoothing forecast with trend adjustment, (α=0.9;β=0.8)

Forecast 2004 (given)	Trend 2004 (given)	α	β
4500	50	0,7	0,4

Given data

Years	Actual sales	Forecast
2004	4410	4500,00
2005	4560	4452,00
2006	4615	4530,84
2007	4725	4601,16
2008	4680	4703,13
2009	4630	4708,34
2010		4666,97

Computing exponential smoothing forecasts

Years	Trend
2004	50
2005	10,80
2006	38,02
2007	50,94
2008	71,35
2009	44,90
2010	10,39

Computing trend

Years	Forecasts(F)	Trend (T)	Forecast with trend adjustment
2005	4452,00	10,80	4462,80
2006	4530,84	38,02	4568,86
2007	4601,16	50,94	4652,09
2008	4703,13	71,35	4774,48
2009	4708,34	44,90	4753,24
2010	4666,97	10,39	4677,36

Computing exponential smoothing forecasts with trend adjustment

Years	Actual sales	Forecast α = 0,7; β = 0,4;	Mean absolute percent Error 100% * (|Error|/Actual demand)
2005	4560		2,13%
2006	4615		1,00%
2007	4725		1,54%
2008	4680		2,02%
2009	4630		2,66%
		Total Error	**9,35%**

Computing forecast mean absolute error (MAPE)

Table 52. Computing exponential smoothing forecast with trend adjustment, (α=0.7;β=0.4)

Forecast 2004 (given)	Trend 2004 (given)	α	β
4500	**50**	**0,22**	**0,09**

Given data

Years	Actual sales	Forecast
2004	4410	4500,00
2005	4560	4519,20
2006	4615	4565,01
2007	4725	4612,75
2008	4680	4674,23
2009	4630	4713,29
2010		4732,09

Computing exponential smoothing forecasts

Years	Trend
2004	50
2005	47,23
2006	47,10
2007	47,16
2008	48,45
2009	47,60
2010	45,01

Computing trend

Years	Forecasts(F)	Trend (T)	Forecast with trend adjustment
2005	4519,20	47,23	4566,43
2006	4565,01	47,10	4612,11
2007	4612,75	47,16	4659,91
2008	4674,23	48,45	4722,67
2009	4713,29	47,60	4760,89
2010	4732,09	45,01	4777,10

Computing exponential smoothing forecasts with trend adjustment

Years	Actual sales	Forecast α = 0,22; β= 0,09;	Mean absolute percent Error 100% * (|Error|/Actual demand)
2005	4560		0,14%
2006	4615		0,06%
2007	4725		1,38%
2008	4680		0,91%
2009	4630		2,83%
		Total Error	**5,32%**

Computing forecast mean absolute error (MAPE)

Table 53. Computing exponential smoothing forecast with trend adjustment, (α=0.22;β=0.09)

6 IMPROVEMENT

The Transportation Modeling and its variants were discussed in paragraph 2.2. The first step of any transportation problem is to transfere the real transportation flow data into transportation matrix. To be able to use transportation modeling methods managers need to know and understand complete transportation flow between their goods suppliers and buyers with current capacity and demands. The second principle of the transportation modeling is to use the right costs minimization approach to find a basic solution of a problem which will help to optimize the cost in the easiest and short-time way. The transportation modeling methods can be used in different sort of business areas with many suppliers, buyers and different quantities. In the stage of strategic planning to achieve an optimum solution and minimize the costs on transportation, managers should consider the real data such as market prices on transportation services, exact figures of needed quantities from all suppliers and total transportation cost of complete routes.

The forecasting methods were discussed in paragraph 2.6. The main issue in solving forecasting problem was to find the smallest forecast error using right forecasting approach which shows the most exact results. Based on results of forecast calculations the author suggests that to achieve the best forecast results with smaller forecast error managers should always consider the numbers of the smoothing constants and try them in different combinations. The best combination of α and β will show the smallest forecast error and helps to achieve the best forecast. The forecast results are very important in business activities and should be also consider in strategic planning as an important factor to perform the process of optimization. Based on results of the paper work and calculations the author of the thesis suggests to provide detailed analysys for the whole existing transportation chain of SMT transport sevices Ltd. Using TORA software and forecasting approaches managers can compute and compare the results of theoretical methods and present expences on transportation.

Research limitations

The author of the thesis has to take in consideration certain limitations. In tansportation modeling calculation part the author did not take into consideration the quantities of the other smaller wood suppliers in Russian territory, because their goods in most cases are for sale in local market. The goods from the smaller suppliers, company uses only in case when demand from the buyers very much exceeds the total supply from the biggest suppliers, which happen very seldom. Another limitation in transportation problem is that the author did not take into concideration such situation when buyer is ready to cover in advance half of transportation expences such as terminal expences and sea freight. Depends on terms of the contract between company and buyer the final prices of the goods may vary and they are different from those that author used in his calculations.

7 CONCLUSION

The thesis was conducted in order to present different transportation modeling methods and forecasting approaches, to analyze the possibility of improvement and integration of transportation modeling methods into existing transportation flow of an existing company operating in Russian market, to minimize the costs on transportation by finding an optimum solution for transportation routes from different production sources to a different points of destination. The use of TORA application was done in order to present that the results of optimum solution for the transportation route of a company, using operation research software are identical to the final results and optimum solution of handmade calculations. All steps and details of optimization, using handmade calculations were done in order to present and describe the full mechanism and details of the transportation modeling methods and also to prove that software is accurate and efficient. The forecasting part was conducted based on real statistical data of existing data and transportaton flow activities in order to show different forecasting approaches and to predict future situation of the company . The author considered the study important, as the minimization costs and optimization of the transportation proccesses may help to improve the company position on the market and increase the profitability of the company by reduction expences on transportation.

8 REFERENCES

Hamdy A. Taha , *Transportation model and its variants , "Operation research 8th edition",* Prentice Hall 2006

Hamdy A.Taha, *Windows-based TORA software and Excel templates to solve transportation problems, "Operation research 8th edition",* Prentice Hall 2006

Hamdy A.Taha, *Transportation modeling, "Operation research 7th edition",* Prentice Hall 2002

Jay Heizer, Barry Render, *Transportation models, "Operations management seventh edition", Prentice Hall 2004*

Jay Heizer, Barry Render, *Forecasting, "Operations management seventh edition", Prentice Hall 2004*

Lee J. Kraevski, Larry P. Ritzman, Manoj K. Malhotra, *Linear programming models, "Operations Management processes and supply chains 9th edition"*, Prentice Hall 2005

Jay Heizer, Barry Render, *The modi and VAM methods of solving transportation problems, "CD tutorial ", Prentice Hall 2004*