Separating feature changes due to damage and environmental conditions using transfer learning

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Abstract
In physics-assisted structural health monitoring (SHM), damage can be identified by model updating to match the experimental data, for example the lowest natural frequencies. A major issue is that natural frequencies are also sensitive to environmental conditions. As damage emerges, it is difficult to determine frequency changes caused by damage alone. In this paper, the two sources of frequency changes are separated using transfer learning or population-based SHM utilizing another structure in the vicinity. A regression model can be built to predict the natural frequencies of the primary healthy structure from those of the undamaged secondary structure. The differences between the actual and predicted frequencies are then supposed to be due to damage. A numerical experiment was performed with two different structures under similar environmental conditions. One of the structures was subjected to damage and the objective was to predict the frequency changes due to damage for a subsequent model updating campaign.

1 Introduction

Structural health monitoring (SHM) can be divided into data-based and physics-based methods. Damage can be detected and possibly localized using measurement data only. In order to acquire more information about damage, for example damage type or extent, a numerical model of the structure, or a digital twin, is needed. To this end, a finite element (FE) model is often built. Once damage is detected, the validated FE model of the undamaged structure is updated to match the new experimental data. The FE model should yield, for example, the same lowest natural frequencies as identified using the monitoring data. Therefore, for successful model updating, the frequency changes due to damage must be measured.

Remark: The features used in this paper are the lowest natural frequencies. Any other damage-sensitive features could also be used, for example mode shapes, but for readability, a more general term “feature” is replaced with a more specific term “natural frequency”.

A major issue in model updating is that natural frequencies are also sensitive to environmental or operational conditions. A simple example is illustrated in Figure 1, in which two lowest natural frequencies are monitored. The blue data points are measurements from the undamaged structure under variable environmental conditions. The red data point is a measurement from the damaged structure. Damage detection is possible using multivariate statistical methods even without measuring the underlying environmental variables, see e.g. [1,2]. However, if the objective is to update the FE model to identify damage, the frequency changes due to damage must be measured. From Figure 1 it can be seen that the frequency changes due to damage are not unique, but there are infinite possible pairs of $(\Delta f_1, \Delta f_2)$, of which three examples are plotted as line segments in the figure. Two of them are obtained using the data mean as a reference (leftmost line, blue) or selecting the shortest distance from the data using principal component analysis (PCA) [3] (middle line, yellow). The true frequency change is the third line segment from the green circle to the red circle, indicating that the measurement was taken at a relatively extreme environmental condition. The green circle represents a data point that would have been acquired if the structure had been undamaged. However, this data point is unknown. Notice that there is no physical justification to use either of the two aforementioned methods to determine the frequency changes due to damage alone. In fact, the two methods yielded an increase of frequency f_1, which is questionable if damage was due to stiffness degradation. The actual consequence of damage was a decrease of both natural frequencies. With this simple example, it was illustrated that there is not enough information to determine the frequency changes due to damage alone, and by using inappropriate methods, incorrect frequency changes as an input to a model updating algorithm may result in faulty identification of damage.

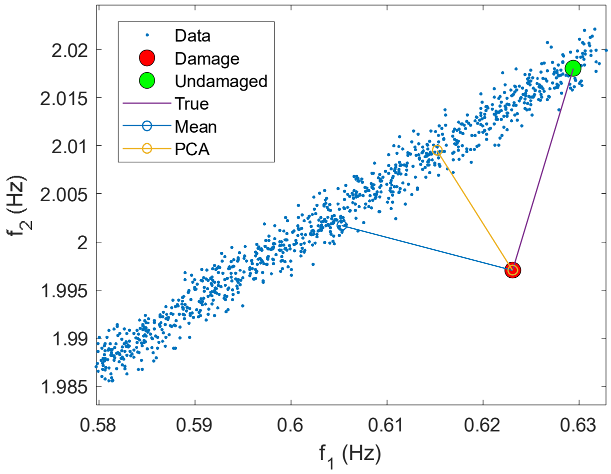


Figure 1: Two natural frequencies influenced by environmental variability. The blue dots are from the undamaged structure and the red circle is from the damaged structure. The three lines are examples of possible frequency changes due to damage.

The objective of the present paper is to obtain estimates of the frequency changes due to damage alone, when training data of the undamaged structure are available under different environmental or operational conditions. Another objective is to investigate if the frequency changes due to damage remain constant at different environmental conditions.

As mentioned above, additional information is needed to obtain the frequency changes due to damage. The most obvious solution is to measure the underlying environmental or operational variables and to build a regression model to estimate the frequencies of the undamaged structure (the green circle in Figure 1). This is also recommended as the first choice. However, this approach may be unsuccessful, if some underlying quantities are unavailable or if the regression function is very complex, e.g. due to non-linear or temporal effects.

A second possible solution to obtain the frequency changes due to damage alone is to use another structure in the neighborhood as a reference. This reference structure is also monitored and assumed to be undamaged and affected by the same environment as the damaged structure. A regression model is built to estimate the natural frequencies of the undamaged primary structure (the green circle in Figure 1) from the natural frequencies of the reference structure.

Because other monitored structures in the neighborhood are utilized to estimate the features of the structure under review, the approach can be classified as population-based structural health monitoring (PBSHM). The main motivation of PBSHM is the lack of available health-state data for supervised learning [4]. It may be possible to transfer label information from one member of the population to another member. The objective is thus to obtain information of the health state beyond damage detection. Therefore, PBSHM, as suggested in [5], is a more comprehensive approach than what is applied in this paper. Also, the relation of the proposed method to transfer learning [6] is discussed.

The paper is organized as follows. Transfer learning and population-based structural health monitoring as applied in this paper are outlined in Section 2. It turns out that the problem reduces to a traditional machine learning problem. In Section 3, the proposed method is applied to numerically simulated structures using linear regression. Concluding remarks are given in Section 4.

2 Transfer learning and population-based SHM

Transfer learning is a framework addressing the problem in which the training data and the test data are from different feature spaces or different distributions [6]. For example, the feature space of the structure *A* may include 10 natural frequencies, while that of the structure *B* includes 7 natural frequencies. Also, the distributions of the features are generally different for the structure *A* and the structure *B*. Both structures, however, share the same latent space, namely the environmental conditions of adjacent structures are similar.

In transfer learning, a domain *D* consist of a feature space *X* and a marginal probability distribution *P*(*X*). A task *T* consists of two components: a label space *Y* and a function $f(\cdot)$ to predict the corresponding label $f(x)$ or $P(y|x)$.

The objective in this paper is to predict the frequency changes due to damage under unknown environmental conditions. Training data are available from the population of undamaged structures under a shared range of environmental conditions. If one structure becomes damaged, its natural frequencies are measured, whereas the frequencies of the corresponding undamaged structure are unknown (Figure 1).

The frequency change vector due to damage $\Delta\mathbf{f}_d$, is

$$\Delta\mathbf{f}_d = \mathbf{f}_d - \mathbf{f}_u \tag{1}$$

where $\mathbf{f}_d$ and $\mathbf{f}_u$ are the frequency vectors of the damaged and undamaged structure, respectively. Because $\mathbf{f}_u$ is not available, it must be estimated. The estimated frequency change due to damage is

$$\widehat{\Delta\mathbf{f}}_d = \mathbf{f}_d - \hat{\mathbf{f}}_u \tag{2}$$

and the estimation error of $\Delta\mathbf{f}_d$ is

$$\Delta\mathbf{f}_e = \widehat{\Delta\mathbf{f}}_d - \Delta\mathbf{f}_d = \mathbf{f}_u - \hat{\mathbf{f}}_u \tag{3}$$

The task is to estimate $\mathbf{f}_u$, which corresponds to *Y* and $P(y|x)$. As discussed before, additional information is needed to this end. The estimation is possible by using another undamaged structure *A* in the neighborhood. Then the source and target domains are the same (e.g. the 10 lowest natural frequencies of the structure *A*). The source and target tasks are also the same, namely a regression function $f(\cdot)$ to estimate *Y*, which is a vector of features of the structure *B* in the undamaged state (e.g. the 7 lowest natural frequencies of the structure *B* if it were undamaged). Because the target and source domains are the same and their learning tasks are the same, the learning problem becomes a traditional machine learning problem [6].

Another approach to transferring knowledge within the framework of PBSHM was suggested in [7], where all points in a base manifold *M* represented different plane trusses. For every point in *M*, there existed another manifold, a fibre *F*, representing different characteristics of the structure due to different environmental conditions or health states. The objective was to estimate the first natural frequency of a truss not included in the training set. A major challenge was to map different structures into points of a manifold.

With some modification, the same approach can be applied to the present problem. Now, the base manifold *M* consists of points representing natural frequencies of the healthy structure *A* at different environmental conditions. Each point thus represents a single environmental condition. For every point in *M*, there exists a fibre *F* representing other structures in the population at the same environmental condition. Consequently, for a structure *B*, the fibre *F* consists of a single point. A mapping between *M* and the fibre bundle is attempted at environmental conditions not included in the training set, which again reduces to multivariate regression.

The present study addressed a problem in which all structures in the population are monitored, and one of them becomes damaged. The objective is to estimate the change caused by damage alone. It is assumed that the environmental conditions are similar for the whole population. This means that the structures must locate in the same neighborhood and are monitored at the same time. Environmental variability is assumed to be much slower than the dynamics of the structures. Further, it is assumed that the same environmental variables, e.g. temperature, humidity, or wind speed, affect the structures in the population. For example, temperature may affect the Young's modulus of the material. The structures need not be similar.

PBSHM is needed only after damage is detected. Assume that damage was detected in the structure *B*. Regression is used to predict the features of the previously undamaged structure *B* by using the features of the other structures in the population. For simplicity, assume that the population comprises only two structures, *A* and *B*. Then, features of the structure *A* are used to predict features of the structure *B* if the structure *B* were undamaged. The features of the undamaged structure *A* could then possibly replace the direct environmental variables in the regression, or to oversimplify, the structure *A* acts as a thermometer.

This is a supervised learning problem. Linear regression is applied in this study for simplicity. The objective is to introduce the main idea and not to focus on details, which could obstruct the big picture. However, it should be noted that the regression is generally nonlinear. Other methods should then be applied, for example neural networks.

Figure 2 illustrates the proposed method. Two structures, *A* and *B*, are monitored under variable temperature *T*. The distributions of the two lowest natural frequencies of the undamaged structures are shown with blue dots. The structure *B* then becomes damaged, and the corresponding simultaneously measured data points are shown with a green circle in Figure 2a and a red circle in Figure 2b. The green circle in Figure 2b corresponds to the data point if the structure *B* were undamaged. This data point is, however, not measured, and must be estimated to obtain the frequency changes due to damage (red line in Figure 2b). The estimation is made by regression using the data from the structure *A* as independent variables and the data from the undamaged structure *B* as dependent variables. The linear minimum mean square error (MMSE) estimation [8] results in the frequencies of the undamaged structure *B*, $\mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{B}}$, given the corresponding data of the structure *A*:

$$\hat{\mathbf{f}}_{\boldsymbol{u}}^{\boldsymbol{B}} = \mathrm{E}[\mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{B}}|\mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{A}}] = \bar{\mathbf{f}}_{\boldsymbol{u}}^{\boldsymbol{B}} + \Sigma_{BA}\Sigma_{AA}^{-1}(\mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{A}} - \bar{\mathbf{f}}_{\boldsymbol{u}}^{\boldsymbol{A}}) \tag{4}$$

and the error covariance matrix

$$\mathrm{cov}[\mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{B}}|\mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{A}}] = \Sigma_{BB} - \Sigma_{BA}\Sigma_{AA}^{-1}\Sigma_{AB} \tag{5}$$

where

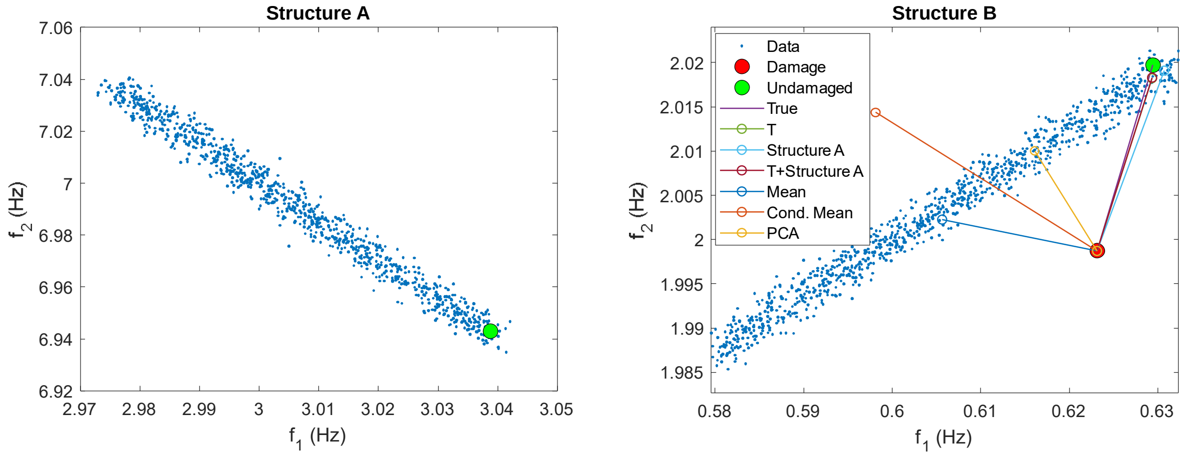
$$\mathbf{f}_{\boldsymbol{u}} = \begin{Bmatrix} \mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{A}} \\ \mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{B}} \end{Bmatrix} \tag{6}$$

and

$$\Sigma = \mathrm{E}\left[(\mathbf{f}_{\boldsymbol{u}} - \bar{\mathbf{f}}_{\boldsymbol{u}})(\mathbf{f}_{\boldsymbol{u}} - \bar{\mathbf{f}}_{\boldsymbol{u}})^{\boldsymbol{T}}\right] = \begin{bmatrix} \Sigma_{AA} & \Sigma_{AB} \\ \Sigma_{BA} & \Sigma_{BB} \end{bmatrix} \tag{7}$$

where E(·) denotes expectation. Here, $\mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{A}}$ can be either (1) the natural frequencies of the structure *A*, (2) temperature *T*, or (3) both the natural frequencies of the structure *A* and temperature *T*. The results can vary depending on the choice of $\mathbf{f}_{\boldsymbol{u}}^{\boldsymbol{A}}$ as can be seen in Figure 2b.

If the measured data of the structure *B* were only used, the results can be very erroneous as can be seen in Figure 2b. For example, if using the mean as a reference, the frequency change vector becomes the blue line. Applying PCA yields the yellow line. Finally, using MMSE, in which each natural frequency of the structure *B* is estimated given the remaining natural frequencies of the same structure, results in the orange line. As can be seen, the frequency changes are far from the true values (red line).



(a) (b)

Figure 2: Two natural frequencies of the structures *A* and *B*. influenced by the same environmental variability. The blue dots are measurements from the undamaged structures and the red circle in b) is from the damaged structure. The green circle in a) is a data point measured at the same time as the red circle in b). The green circle in b) is the corresponding data point (unmeasured), if the structure *B* were undamaged. The lines are examples of possible frequency changes due to damage.

3 Numerical experiment

A numerical experiment was performed with two different structures in the population. The structures were located in the same environment and monitored simultaneously. One of them became damaged with an increasing damage level after 600 measurements, while the other structure remained undamaged. The objective was to estimate the changes of the lowest natural frequencies due to damage alone for a subsequent model updating assignment.

The objective was not to simulate the whole monitoring process starting from the sensor data, but the lowest natural frequencies were obtained from the finite element models simply by using eigenvalue analyses. No noise was added to the computed frequencies.

3.1 Building (frame)

The structure *A* was a concrete frame with three floors and three columns (Figure 3). The total width was 12 m and the height was 12 m. All beams and columns had a 300 mm × 300 mm square cross section. The material density was $\rho = 2500$ kg/m^3. Only the columns were influenced by temperature, while the horizontal beams had a constant Young's modulus of $E = 40$ GPa. The columns were fixed at the bottom. Damage was located in a single beam element with a length of 1.0 m (Figure 3). It was a decrease in the height of the cross-section from 300 mm to 290 mm, 280 mm, 270 mm, 260 mm, and 250 mm.

The outside temperature in each measurement was random between –25°C and +40°C. The column temperature was equal to the outside temperature plus a random Gaussian term $N(0, 0.2)$ °C. All columns were at the same temperature. The relationship between the Young's modulus E and temperature T (°C) was piecewise linear (see Figure 5a):

$$E = \begin{cases} E_0 + k_1 T, & T < 0 \\ E_0 + k_2 T, & T \geq 0 \end{cases} \tag{8}$$

where $E_0 = 40$ GPa, $k_1 = -0.6$ GPa/°C, and $k_2 = -0.125$ GPa/°C.

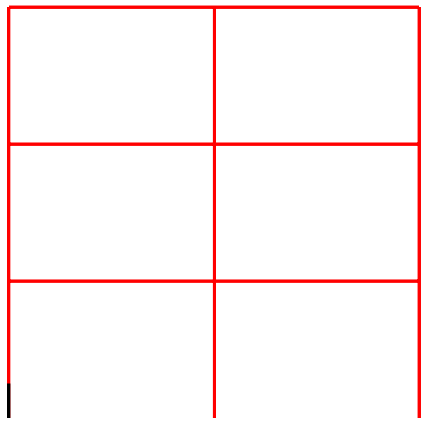


Figure 3: Structure *A*: a concrete frame. Damage location is shown in black.

3.2 Bridge

The structure *B* was a bridge deck with a concrete slab and steel stiffeners (Figure 4). The bridge was 30 m long and 11 m wide. More details can be found in [9].

The outside temperature was the same as with the structure *A*. The bridge ends were at random temperatures equal to the outside temperature ± 1°C. The temperature distribution between the ends was linear, except for Gaussian noise $N(0, 0.2)$ °C, which was added to the temperature values in each element row.

Temperature variation affected only the Young's modulus of the concrete slab according to (see Figure 5a):

$$E = \begin{cases} E_0 + k_3 T, & T < 0 \\ E_0 + k_4 T, & T \geq 0 \end{cases} \tag{9}$$

where $E_0 = 40$ GPa, $k_3 = -0.8$ GPa/°C, and $k_4 = -0.25$ GPa/°C. Sample distributions of the Young's modulus are shown in Figure 5b.

Damage was an open crack in a steel girder with an increasing severity (Figure 4).

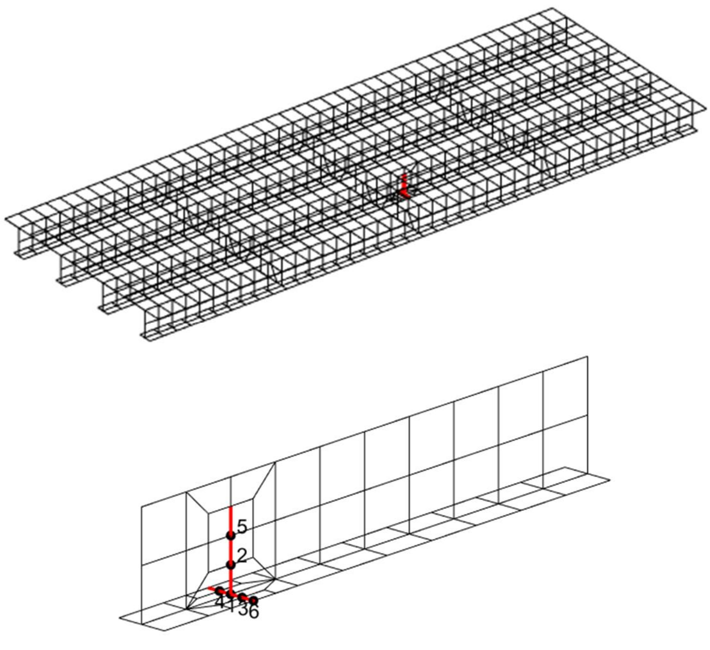
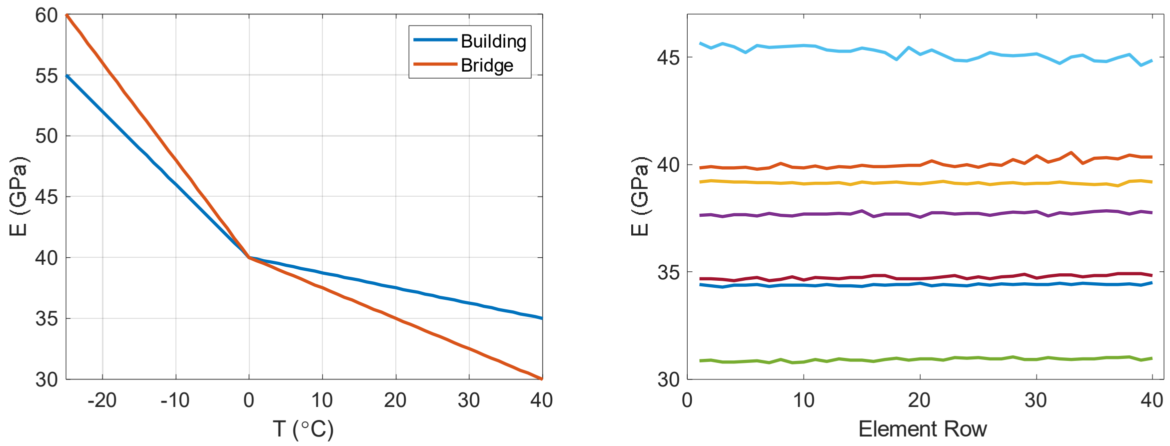


Figure 4: Structure *B*: a bridge deck and a detail of a girder with an open crack. Damage (a crack) is plotted in red. The numbers indicate the order of damage evolution.



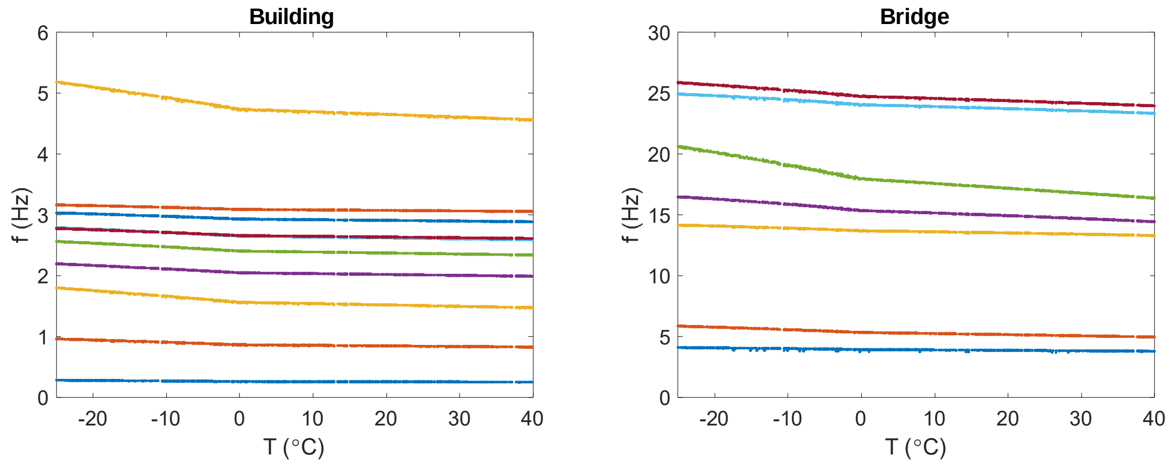
(a) (b)

Figure 5: a) The relationships between temperature and Young's moduli. b) Sample distributions of the Young's modulus along the bridge.

3.3 Data

The data were the ten lowest natural frequencies of the frame and the seven lowest natural frequencies of the bridge. They are plotted in Figure 6 as a function of the outside temperature. Note a slight variability due to the assumed randomness outlined earlier.

An additional pre-processing step was needed for the frame structure, because the order of modes 6 and 7 interchanged at around –8°C. Mode pairing was therefore necessary. The overlapping of frequencies 6 and 7 can hardly be distinguished from Figure 6a.

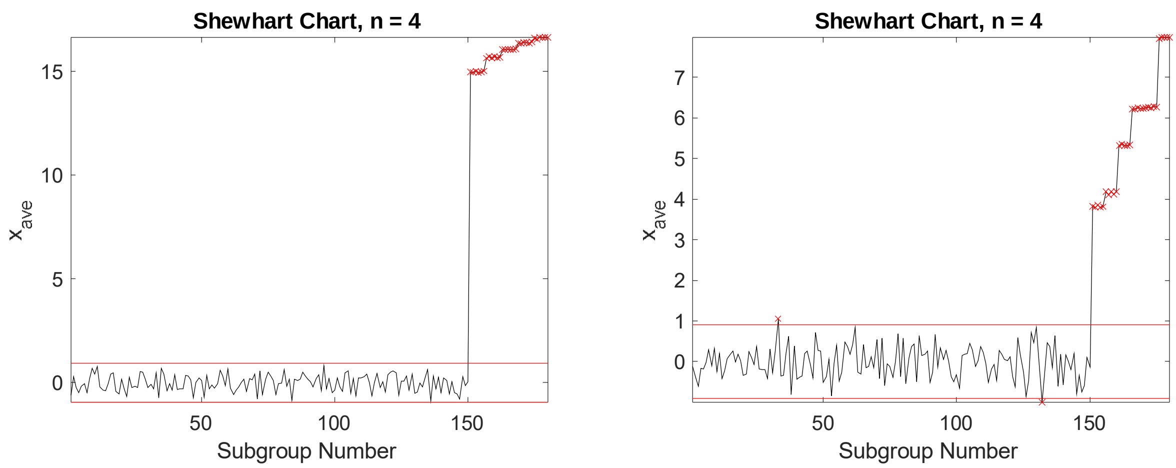


(a) (b)

Figure 6: The lowest natural frequencies as a function of the outside temperature.

3.4 Damage detection

The first step was damage detection, which could be done with the data from the monitored structure only. Damage detection was performed using Shewhart, or xbar, charts with a subgroup size of 4. Environmental influences were taken into account by applying whitening transformation. More details of the damage detection algorithm can be found in [9]. Control charts for the frame and bridge are plotted in Figure 7. Logarithmic scaling was applied for clarity. All damage levels are clearly distinguishable. Note that no noise was added to the computed natural frequencies yielding too optimistic damage detection performance.



(a) (b)

Figure 7: Damage detection: a) building, b) bridge.

3.5 Frame damage

Once damage was detected, the frequency changes due to damage alone were estimated, which is the main objective of this paper. Different methods were applied to this end.

First, the frame structure was damaged with five different severities (Figure 3). The bridge remained undamaged during the whole process. All measurements were taken at different environmental conditions. The actual changes of the first ten natural frequencies due to damage in percentage terms are plotted in Figure 8a. All frequencies decreased as expected, because the stiffness of the damaged element decreased. Note that the damage effect was not constant at different temperatures, but had slight variability. Therefore, it is not feasible to strive for overly accurate frequency changes for model updating.

Frequency changes due to damage were estimated using the seven lowest natural frequencies of the undamaged bridge measured at the same environmental conditions as the frame. A linear regression model was identified using data from the two structures when both were undamaged. The regression model was not very accurate as can be seen in Figure 8b for measurements 1–600. The same regression model was applied after the frame was damaged and the results are shown in the same figure for measurements 601–720. Although the results seem quite noisy, a correct trend can be observed. All natural frequencies decreased on average. Also, the magnitudes of the frequency changes were correctly estimated, at least on average. The increasing damage levels could also be tracked.

Note that the absolute frequency changes due to damage were very small. Even the largest damage level resulted in an absolute maximum change of only 0.023 Hz for mode 10. The maximum root mean squared error (RMSE) of the estimated frequency change was 0.006 Hz, also for the frequency of mode 10.

As a comparison, the frequency changes were also estimated using the data from the damaged structure only by applying PCA or MMSE. The frequency change estimates are plotted in Figure 8c for PCA and Figure 8d for MMSE. Although quite precise, the frequency changes were incorrect, especially for the frequencies predicted to increase due to damage.

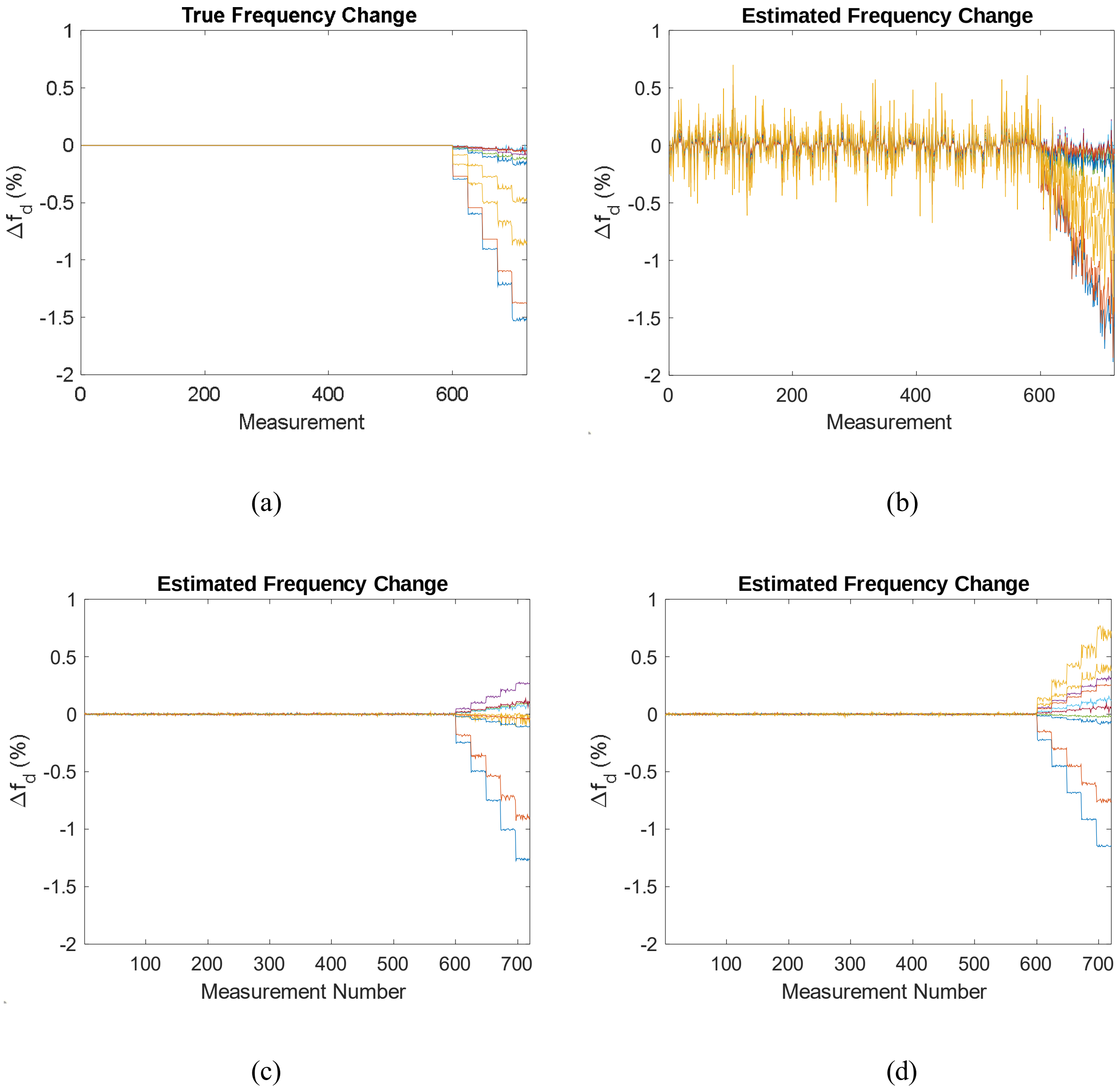


Figure 8: Changes of natural frequencies of the frame due to damage alone. a) true, b) estimated using the bridge data, c) PCA, d) MMSE.

3.6 Bridge damage

Next, the bridge was damaged, while the frame remained undamaged. The objective was to estimate the frequency changes due to damage alone. Six damage levels occurred (Figure 4), and the true frequency changes of the seven lowest modes in percentage terms are plotted in Figure 9a. Again, the frequency changes were not constant, but depended also on the temperature suggesting that it is not worth pursuing too accurate frequency changes. All frequencies decreased as expected.

A linear regression model was built between the natural frequencies of the two undamaged structures at unknown temperatures. The model was fixed and applied to all data before and after the bridge was damaged. The results are shown in Figure 9b. Measurements 1–600 were acquired from the undamaged structure, and measurements 601–720 from the damaged structure. Although the results were quite noisy, a correct trend can be observed. All natural frequencies decreased on average. Also, the magnitudes of the frequency changes were correctly estimated, at least on average. The increasing damage levels could also be tracked, yet not too clearly. Only the change of the frequency of mode 1 was estimated quite accurately, because the absolute change was relatively large, 0.18 Hz for the highest damage level, and the RMSE of the corresponding frequency estimate was only 0.003 Hz (blue line in Figure 9b).

The frequency changes were also estimated using the data of the damaged structure only by applying PCA or MMSE. The frequency change estimates are plotted in Figure 9c for PCA and Figure 9d for MMSE. PCA predicted the largest frequency drop quite accurately. However, the predicted frequency changes would probably result in false identification of damage in model updating, because some frequencies were predicted to increase due to damage. MMSE yielded even more unrealistic results.

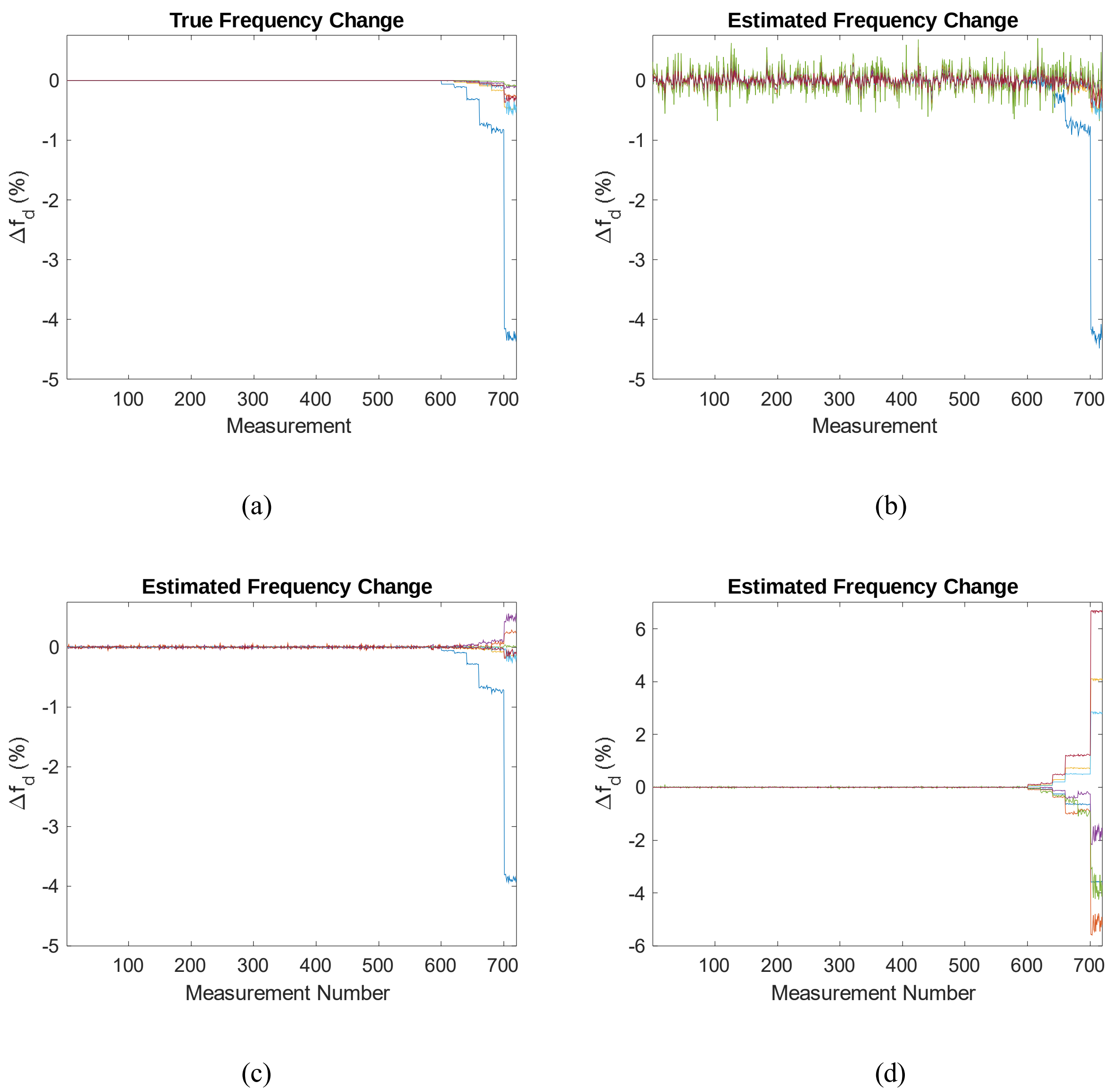


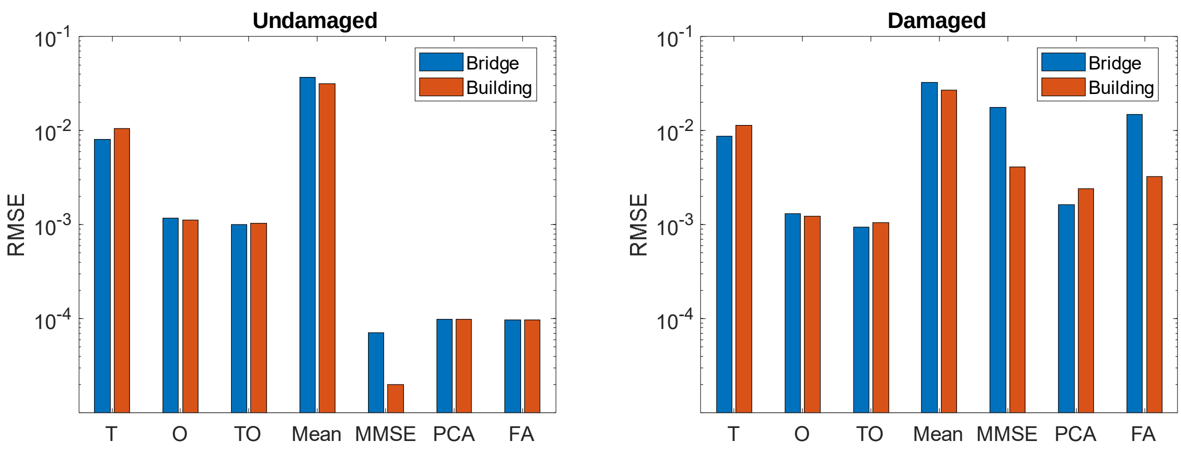
Figure 9: Changes of natural frequencies of the bridge due to damage alone. a) true, b) estimated using the frame data, c) PCA, d) MMSE.

3.7 Validation

Finally, the proposed method to estimate the frequency changes due to damage alone was evaluated by computing the root mean square errors (RMSE) of all frequency change estimates relative to the corresponding undamaged frequencies. The frequency changes due to damage were estimated separately for the frame and for the bridge, while the other structure remained undamaged. Different methods were compared both for the undamaged and damaged structure under variable temperature. The RMSE values are plotted in Figure 10. The following methods were applied. (1) The outside temperature *T* was used as the independent variable (symbol T); (2) The frequencies of the other structure were used as independent variables (symbol O); (3) Both the outside temperature and the frequencies of the other structure were used as independent variables (symbol TO); (4) The mean frequencies of the structure under review were used as a reference (Mean); (5) The frequencies were estimated using the remaining frequencies of the structure under review (MMSE); (6) Principal component analysis (PCA); (7) Factor analysis (FA) [10]. Methods 1–3 used additional information, whereas methods 4–7 used only the frequencies of the structure under review.

From Figure 10a it can be seen that MMSE, PCA, and FA yielded the most accurate estimates for the undamaged structure. Using the outside temperature only was not very successful, but the most inaccurate results were obtained by subtracting the mean from the measured value.

For the damaged structure, which was the target case, the situation changed. Using the information of the structure under review only, the estimates became substantially more inaccurate (MMSE, PCA, and FA) as expected. Notice the logarithmic scale. By using additional information (outside temperature and/or the frequencies of the other structure), the estimation errors did not change. The most accurate frequency change estimates were obtained using the information of the other structure and the outside temperature (Figure 10b). However, the advantage of including the outside temperature was not significant, and using the frequencies of the other structure only yielded almost the same accuracies. For either structure, the total RMSE was about 0.001, or 0.1%, when using PBSHM, which is acceptable. However, the accuracy would decrease in practice due to measurement error and more complex environmental effects. Nevertheless, showing the performance of PBSHM compared to the single structure approach is a promising result.



(a) (b)

Figure 10: Root mean square errors of the estimated relative natural frequency changes due to damage alone using different approaches. a) undamaged, b) damaged.

3.8 Conclusion

A method was proposed to estimate the feature changes due to damage alone under variable and unmeasured environmental or operational conditions. The accuracy of the estimate is important in model updating to identify damage.

Damage detection is possible by monitoring the changes of the damage-sensitive features of the structure e.g. using regression. On the contrary, once the structure is damaged, the regression model is no longer valid, and the current information is not enough to predict the feature changes caused by damage. It was proposed to utilize other monitored structures in the same neighborhood and apply regression to predict the features of the damaged structure if it were undamaged. The features of the damaged structure are measured, while those of the corresponding undamaged structure are estimated. The difference is the damage effect, which can be used in model updating.

Because the source and target tasks were the same, and the source and target domains were the same, the transfer learning problem became a traditional supervised machine learning problem. Linear regression was applied in this paper for simplicity, but any supervised machine learning technique could be applied. Two quite different structures were included in the population in the numerical experiment. The main observations are: (1) The regression yielded correct changes on average; (2) The estimation error was quite large; (3) Using the features of the damaged structure only and applying PCA or MMSE yielded small variance but incorrect mean values; (4) The frequency changes were not constant but depended on the environmental variables; (5) Temperature alone as a dependent variable was not successful due to non-linear effects; (6) Damage detection was possible using the features of the current structure only; (7) Regression was possible with heterogeneous structures in the population.

Damage identification by model updating using the estimated feature changes was left for future studies. Experimental research is also necessary.

Acknowledgements

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