



jamk.fi

Development of Materials Management System

Case Black Bruin

**Focus on Inventory Replenishment
system and Modelling of costs**

Anton Saukkonen

Bachelor's Thesis

May 2017

School of Technology, Communication and Transport

Degree Program in Logistics Engineering

Jyväskylän ammattikorkeakoulu

JAMK University of Applied Sciences

Description

Author(s) Saukkonen, Anton	Type of publication Bachelor's thesis	Date Month Year
	Number of pages 146	Language of publication:
		Permission for web publication: x
Title of publication Development of Material Management System Case Black Bruin		
Degree programme Logistics Engineering		
Supervisor(s) Sipilä, Juha Lehtola, Pasi		
Assigned by Black Bruin Inc.		
<p>Description</p> <p>This thesis work was assigned by a manufacturing company called Black Bruin Inc. BB Inc. is the world leading supplier of radial piston hydraulic motors and rotators. The aim of this study was to develop the company's materials management system, which included the inventory control and costing model. In addition, the optimization of the total inventory cost was implemented regarding the optimal values of the decision variables. During the research process, the author used the mixed research method, which included elements of both qualitative and quantitative methodologies. The core data for the study was obtained from a literature review, interviews, observations and case-study examples. Based on the collected information, the author identified the most appropriate inventory control solution for the Black Bruin Inc. case and constructed it in Excel by using VBA (Visual Basic for Applications) code features. Furthermore, the constructed model included a costing model where the elements of the total inventory cost could be seen, analysed and calculated on a common or separate basis.</p> <p>As a result, the constructed model could be used as a tool in the decision-making process regarding the inventory operations. The model can compute the optimal order quantity and reorder point decision variables regarding the minimum objective value of the total cost. Moreover, the model calculates the values of inventory performance measures and allows adjustment of the decision variables, indicating the effect of change on performance measures and the total cost of inventory. Furthermore, the model also includes a sensitivity analysis which emphasizes the most uncertain input-data elements and allows their alignment.</p> <p>Finally, the study as well as the constructed model were analysed, and the essential outcomes derived. At the end of the research process, the author outlined the advantages and disadvantages of the model as well as its most sensitive and uncertain elements. In addition, guidelines for the model's usage, suggestions for improvements and further research recommendations were given and thoroughly explained.</p>		
<p>Keywords (subjects)</p> <p>Materials Management, Inventory Operations, Inventory System, Mathematical Modeling, Operations Research, EOQ Model, Base Stock Level Model, The (Q, r) Model, VBA Programming</p>		
Miscellaneous		

1.	Introduction.....	4
1.1	Motivation and Background for research	5
1.2	Company Description	5
1.3	Materials Management at Black Bruin Inc.	6
2.	Research methods.....	7
2.1	Qualitative research method	8
2.2	Quantitative research method	10
2.3	Mixed research method	11
2.4	Methods of collecting data.....	13
2.5	Research questions, focus and objective	17
2.6	Limitations of the research.....	18
2.7	Overview of research structure.....	20
3.	Literature review	23
3.1	Introduction to Materials management.....	23
3.1.1	Importance of materials management in the industrial business	24
3.1.2	Objectives of Materials Management.....	25
3.1.3	Primary Functions of Materials Management	26
3.2	Inventory	29
3.2.1	What is Stock and Inventory?.....	29
3.2.2	The reasons for holding inventory	31
3.2.3	The role of inventory	33
3.2.4	Inventory strategies.....	34
3.2.5	Lean Inventory Management.....	39
3.2.6	Just-In-Time Inventory.....	40

	1
3.2.7 Inventory components	42
3.2.8 Costs of Inventory	50
3.2.9 Types of Inventories based on the demand pattern.....	57
3.3 Inventory control.....	60
3.3.1 Inventory policies	60
3.3.2 ABC Analysis	64
3.3.3 Economical Order Quantity model.....	69
3.3.4 Statistical Inventory models.....	74
3.3.5 Base stock level model	74
3.4 The (Q, r) Model	83
3.5 Mathematical Modeling.....	101
3.5.1 Optimization.....	103
3.5.2 Sensitivity analysis.....	104
4. Research implementation	105
4.1 Process development of the model	105
4.2 Problem definition.....	108
4.3 Summation of the collected data	109
4.4 Construction of the model concept	112
4.5 Development of the model	116
4.5.1 Why VBA?.....	117
4.5.2 User Interface Design	118
4.5.3 Estimation of the input variables for the costs	118
4.5.4 Optimization.....	120
4.5.5 Sensitivity Analysis	121
4.6 Model Validation	122

5.	Discussion and Analysis of results	124
5.1	Analysis of constructed model	124
5.1.1	Conceptual vs. Executable version	124
5.1.2	Robustness of the results	125
5.1.3	Advantages and Disadvantages of the model	128
5.1.4	Sensitivity of the model.....	129
5.2	Discussion of research questions	130
5.3	Advices for Management	136
5.4	Suggestions for further research and improvements	138
6.	Conclusion	139
	Acknowledgements	140
	References.....	141
	Appendices.....	146
	Figure 1. MRP II Hierarchy example obtained from Hopp & Spearman (2008, 140).....	27
	Figure 2. Scope of materials management.	29
	Figure 3 The role of inventory in Operations Management.	34
	Figure 4 Cycle stock graph (Adapted from Waters 2003, 6-7).....	43
	Figure 5 Safety stock vs Service level (King 2011, 33-35).....	45
	Figure 6 Normal distribution according to Gauss (Hoppe 2008, 373)	47
	Figure 7. Total Cost of Inventory distribution according to Vermorel (2013) visualized by the author.	54
	Figure 8 Continuous Review Policy vs Time. (Vrat 2014, 29-33).....	61
	Figure 9 Periodic Review Policy vs Time (Vrat 2014, 29-33)	62
	Figure 10 Optimal Review Policy (Vrat 2014, 29-33)	63
	Figure 11 Typical ABC Curve showing the 80/20 relationship. (Jessop, Morrison 1994, 157).....	65

Figure 12 ABC Analysis classification according to Value and Frequency according to Gwynne (2011, 62-64).....	66
Figure 13. Item categories in make-to-order assembly system according to Spearman (2015)	68
Figure 14 Inventory versus time in EOQ model. (Hopp & Spearman 2008, 51)	70
Figure 15 Costs' functions in EOQ model. (Hopp & Spearman 2008, 52)	71
Figure 16 Inventory position in Base Stock level model. (Hopp & Spearman 2008, 73).....	76
Figure 17 Inventory position in the (Q, r) model. (Hopp & Spearman 2008, 79).....	84
Figure 18 Expected Inventory position in the (Q, r) model. (Hopp & Spearman 2008, 83).....	91
Figure 19. Operations Research Process by Rardin (2017, 3)	102
Figure 20 Development process of the model.....	107
Figure 21 Initial user interface design of backorder's calculator.....	113
Figure 22. Initial User Interface Design of database for calculated items. ...	114
Figure 23 Initial User Interface Design of costing calculator.	115
Figure 24 Preliminary algorithm of the model's usage.	116
Table 1. The difference between Qualitative and Quantitative research method.	12
Table 2 Research structure.....	22
Table 3. Correlation between desired service level and Z-Score.....	47
Table 4 The ABC Analysis nine categories. (Adapted from Vollmann 1997, 722-725)	66
Table 5 Classification of Inventory Models and Coincidence with BB Inc. ...	111

1. Introduction

Nowadays, materials management is one of the most essential functions in almost any type of industry. The reasons behind it are numerous. However, this fact could be well explained by the current situation in the industrial world. Due to the dramatic technological progress over the last thirty years and the invention and development of absolutely new and innovative products or technologies, the markets are becoming more and more complicated not only for the small and medium enterprises but also for the giant industry leaders. Nevertheless, the competition is growing exponentially, and we can continuously behold the emergence of new and powerful market players. At this point, one might ask how companies could stay competitive in such a harsh and emulative environment. The answer is unambiguous: nowadays, an excellent performance is reached not only through the supply of high-quality products but also through the continuous enhancement of operations inside the companies and supply chains. The well-known approaches that are intended to develop the problematic issues are related to the science of materials management and operation research. According to it, the importance of materials management has undoubtedly a crucial meaning for strategic improvement, competitive advantage and operational excellence of the companies and whole supply chains. (Magad & Amos 1995, xi-xii)

At the moment, extensive research has been carried out in the area of MM. The variety of methodologies concerning the control of material flows has been developed in all the areas of MM. Consequently, the implementation of various approaches to the development of MM functions is indeed a must, and it is required for the sake of staying competitive. (Magad & Amos 1995, xi-xii)

1.1 Motivation and Background for research

The project was assigned by Finnish manufacturing company Black Bruin Inc. The main request of the company was the development of a materials management approach that would help to clarify and control the issues concerning the management of materials and the costs related to these activities. The motivation for the assignment provided by the BB Inc. referred to the fact that during the course of this study, BB Inc. was developing a new product called "On-Demand Control System," which involved a significant portion of components that the company was not able to manufacture. Consequently, these components were supposed to be supplied from outside sources and, therefore, operational and cost policy for the control of this function had to be developed. Therefore, the main purpose was to conduct research in the area of MM and inventory control and use the obtained information to develop an MM system that would fit the BB Inc. case and provide an optimal flow of outside-manufactured components, control the costs related to the performance of such activities and efficiently support production operations.

1.2 Company Description

BB Inc. is a manufacturing company that was initially established in 1959 with the brand name Sisu Auto Oy. Nowadays, BB Inc. is considered to be one of the most progressive manufacturers of radial piston hydraulic motors and hydraulic rotators. The products of BB Inc. are widely used in many industrial areas, such as forest industry, construction and mining, agriculture, road building and other areas where vehicles are supposed to operate in difficult terrain conditions.

BB Inc. employs around 100 workers cumulatively, and 80% of its products are sold outside of Finland. Generally, BB Inc. has its offices and distribution

network covering 25 countries, and the majority of the manufactured goods are exported to North America, South Africa, Sweden, the United Kingdom, China, Germany, Belgium, the Netherlands, and many more.

BB Inc. has a strongly developed R&D department with experienced engineering professionals and its own testing laboratory where they are able to test their products by simulating different operating conditions.

Moreover, the R&D department closely cooperates with the customers and brings advanced information about customer requirements into the specificity of their expertise. This makes the products of BB Inc. unique and robust in the particular market niche.

Furthermore, due attention in the company is paid to quality assurance.

Since the year 1993, BB Inc. has obtained the globally certified quality management system standard ISO 9001. Moreover, since 1999 the operations of the company have corresponded to the environmental quality management standard ISO 14001.

Due to the scope of its operations, the BB Inc. is referred to as an SME type of an organization. Despite this, its hydrostatic transmission, motors and rotation solutions are known as some of the most innovative and reliable products in this industrial area.

1.3 Materials Management at Black Bruin Inc.

The majority of logistics functions, such as distribution, transportation, and warehousing are outsourced at BB Inc. The transportation processes are implemented by transportation providers over local areas, while distribution and warehousing are initiated by the dealers, which are responsible for certain geographical areas, such as a region or a country. However, the main manufacturing facility in Jyväskylä has its own warehouse that is used to support the main manufacturing operations.

The basic control of materials management functions is initiated through the procurement department of the company. In particular, the procurement department is the only unit in the organization, which is responsible for the control of the supplied materials. The main assignments of the procurement department are to maintain supplier relationships, initiate purchasing processes of the materials from outer resources and control the amounts of money spent in purchasing. In addition, the functions related to inventory control are also performed by the procurement department.

2. Research methods

Research is the way to provide answers to questions which arise during the study of a certain subject. Research methods are special techniques that are used to collect, sort and analyze data and make certain conclusions that then could be used as an output or result in a certain process. (Walliman 2011, 22) Before any potential research project is carried out, it is very important to determine the appropriate research method for the particular case of study. The relevance of the research method is very important since it will affect the whole process of study and hence the validity of the results. According to Dahlberg & McCaig (2010, 14), research should be based on empirical data and previous research. In addition, it should be objective, valid, reliable and allow to make generalizations.

There are three main methods that could be used in research: the qualitative and quantitative as well as the mixed method which is a combination of the two previous ones. Obviously, all methods cannot be viewed as separate methodologies, and conversely, they constitute the opposite parts of a continuum. Usually, a study requires either a qualitative or quantitative analysis, depending on the subject-matter. However, very often a topic cannot be analyzed from only one perspective and in that case, it is sensible

to use the mixed research method which is an intersection of two traditional methods. (Creswell 2014, 32)

2.1 Qualitative research method

One of the most common properties related to qualitative research is that the collected, sorted and analyzed data is interpreted in the form of words in all ways (written and spoken). Moreover, the analyzed data is usually presented in an unstructured form. The process of collecting information includes the implementation of interviews, using questionnaires, observation procedures, reading and analyzing different sources of literature and other documents. The collected data may be interpreted in terms of local meanings and information collected in a specific in context. Yet, it might contribute to a more general understanding of the topic. The interpretation of information has a narrow and specific focus, but at the same time it provides deeper and detailed descriptions of the issue. Research methodology seeks for commonalities between different patterns of data, but at the same time tends to explore and analyze differences between them. Another important property related to a qualitative analysis is the emotional involvement/engagement of the researcher. Since the most common source of information is words, the researcher is more likely to bring in his own interpretations of the issue that might affect the result. In addition, a qualitative method requires more time to complete the research process. As analyzed data is unstructured and there is no certain algorithm, the researcher has to determine those procedures himself. (Braun & Clarke 2013, 4)

Advantages and disadvantages of qualitative research

The qualitative research methodology has its advantages and drawbacks. The main advantage of the method is that collected data is analyzed with

more in-depth and detailed techniques, which results in a more profound output. Secondly, the frameworks of the data collection methods in qualitative research, such as interviews, are not restricted by a rigid form, and the researcher can guide the discussed issues and interviewee in real time, while in quantitative research the fixed framework restricts the 'direction' of the collected information. Furthermore, the nature of the collected data in qualitative research is based either on personal experience or on experience gained from real cases that were developed in the past and now exist as a rule or axiom. According to this view, qualitative data may be more cogent and powerful when comparing to the quantitative one.

(Advantages and Disadvantages of Qualitative Research 2014)

Despite the strengths of the qualitative method, it also has its disadvantages. The main issue concerning the drawbacks is that the quality of the research is highly influenced by the skills of the researcher. Moreover, the quality could also be affected by personal the engagements/interpretations of the person conducting the study. As it was mentioned earlier, the usage of a qualitative method may sometimes presume the personal involvement of the researcher and influence the objectivity of the findings, which will be critical for the final output as it may produce wrong results. The next issue is that results gained by using a qualitative method may sometimes be very difficult to replicate and apply to another case, because the nature of the study very often has a narrow and specific focus. In addition, due to the nature of techniques, data collection and data analysis methods in qualitative research are much more time- and labor-consuming than with quantitative methods. Hence, the implementation of the study requires more resources. (Advantages and Disadvantages of Qualitative Research 2014)

2.2 Quantitative research method

Apart from qualitative methodology, a quantitative analysis uses data presented in the form of numbers. The data under analysis is usually structured, but the amount might be much bigger when comparing to qualitative data. The analysis is implemented using mathematical and statistical operations, models, tools, formulas etc. Very often, these could be done by using the computer with certain software. (Walliman 2011, 113)

The analyzed information is intended to determine the relationship between random variables, identify their correlations and explains how the findings are related to the certain population of the data. The interpretation of the results covers a broader scope than in the qualitative approach, but at the same time, it does not provide such a deep and complex analysis of each topic separately. Instead, the results give a basis for more common and general conclusions. The output of a quantitative analysis is to find a certain consensus or general pattern, which will simplify the difference between statements to an average statement. Unlike in the qualitative methodology, a quantitative analysis does not assume emotional involvement of the researcher. On the other hand, it values personal detachment and objectivity. Hence, the researcher is unlikely to include his own interpretations into the survey, and the results are impartial. Since the data is structured, and the analysis is very often implemented with help of software or a computer, the whole process can be completed more quickly than with the qualitative methodology. (Braun & Clarke 2013, 4)

Advantages and disadvantages of quantitative research

Quantitative research methodology has also its strengths and drawbacks. First of all, the main advantage of the quantitative method is that it works with quantifiable data, which is usually presented in the form of numbers.

However, at the same time, the method excludes the usage of such data collection methods as an interview where the personal involvement of the researcher might play a crucial role. Moreover, the data analysis and processing methods often include the use of mathematical operations, models and tools, which generates more common but more impartial results. According to this, the overall output of the quantitative analysis is considered more objective and reliable. Secondly, a quantitative analysis deals with large quantities of data and produces more general, common result than the qualitative. Consequently, the results are not as unique and complex as with the qualitative method, but they are more standardized, and the output could be easily replicated to other similar cases. In addition, a quantitative analysis is less time- and labor-consuming, as it often uses surveys for data collection and computers with software for data analysis. (“DME for peace: Section 1.3 Quantitative Research Module”)

However, the quantitative method also has its shortcomings. Firstly, quantitative research is capable of dealing with large samples of data at a time, but the produced results are too general and fully reliant on the quality of the analyzed data. If the survey consists of irrelevant questions, then it could be interpreted in a wrong way and, hence, produce wrong results. The distance between the researcher and people in a survey could have a negative impact, because the matter of the survey might be interpreted wrongly or misunderstood. As a result, the quality of the output might be partly or fully irrelevant. (“DME for peace: Section 1.3 Quantitative Research Module”)

2.3 Mixed research method

The mixed research method is an approach that uses both methodologies: qualitative and quantitative. Unsurprisingly, mixed research is the most commonly used method nowadays, since the problems of surveys are not absolutely related to any specific discipline, so that it is often impossible to

exclusively apply either a qualitative or quantitative analysis. The mixed method uses data that can be interpreted in both ways: words and numbers. Furthermore, the properties of the mixed research method are not 'fixed' and may vary depending on the subject-matter: sometimes a problem requires a more qualitative analysis than quantitative and vice versa. (Creswell 2014, 32)

Adapted from: (Braun & Clarke, 2013, 5)

Table 1. The difference between Qualitative and Quantitative research method.

	Qualitative	Quantitative
Form of data for analysis	Words	Numbers
Scope of research	Narrow, but more detailed	Broad, but more general
Form of interpretation	Descriptive	Analytical
Research approach	Theory-generating, inductive	Theory-testing, deductive
Relation to researcher	Researcher emotional involvement	Researcher emotional detachment
Design of Research	Less fixed and indefinite	More fixed and predefined
Execution time	Takes longer time, because of interpretation and lack of algorithm	Can be completed quicker, due to structured data and availability of formula

2.4 Methods of collecting data

General differences in value of data.

According to Westat (2002, 43), qualitative and quantitative methods of research provide a compromise between the width and deepness of the study. While a quantitative research method provides more generalization and commonality in outputs about a certain population of data, a qualitative study method is focused on providing a more specific and targeted analysis of the information. According to the nature of study, different methods of data collection are used. However, when the subject requires the use of mixed research expertise, it is good to utilize a combination of the different techniques.

Interview is a standard method of data collection. This method is initiated as a conversation between the researcher and the person who might possess the information needed for the research. The researcher assumes that the perspective of an opposite participant is meaningful, knowable, can clarify the discussed issue and affect the result of research implementation. Usually, there are two types of interviews that could be used in a study: structured and in-depth interviews. A structured interview presupposes the usage of a certain rigid questionnaire that will carefully 'guide' the researcher through the conversation. An in-depth interview does not presume the usage of any rigid structure and allows the researcher some sort of 'free-riding' during the conversation. (Westat 2002, 50) Interviews can be used almost at any stage of the research processes, and they have their advantages and disadvantages. The main advantage of the interview is that the researcher can acquire knowledge and information straight from the source and eliminate any misunderstanding of details right at the moment of the conversation. The main disadvantage is that the information collected

from the interviewees might be based on the specific opinion of the person and, hence, might be very subjective. Due to this, the collected data should not be considered an 'absolute truth' and should be critically analyzed afterward.

Observation is a method of data collection when the researcher gathers information straight from the source by observing the processes, program or behavior being studied. This method is claimed to be very accurate and precise due to its nature that assumes continuous and long-term study of certain behavioral and process models. By observation of operations and behaviors, the researcher is capable of developing a holistic picture of behavioral models, processes, and operations during a certain time-scope and under certain conditions. Moreover, the observation of processes from a side-perspective may identify deep understanding of the context issue, which the observed participants were unaware of or what they were not willing to discuss. (Westat 2002, 53) The main advantages of the observation method are that it provides precise information concerning the studied process or behavioral model and allows the researcher to be involved in the natural context of the studied situation. The outcomes might be surprisingly new and innovative for the researcher and at the same time they form a holistic, corresponding to the truth, perspective, which will provide genuine information about the studied issue. However, the disadvantages of this method are also significant. First of all, the observation method is a time-consuming one as it requires constant physical presence of the researcher. Secondly, in order to receive genuine and relevant information, it requires the involvement of highly trained professionals or even experts of the studied issue, as beginners or amateurs may miss important points or make irrelevant emphases during the observation periods. Finally, often it might affect the behavior of the participants and, hence, the result of the study.

The observation method could be best used during the formative and conclusive stages of a study. (Westat 2002, 55)

Document and literature studies are another widely used method of data collection. It is frequently claimed that this method is the most widely used nowadays among researchers from different fields. The method might be initiated in two ways or divided into two categories: a study of public records and study of personal documents. Public documents include almost all documents that are written and provided for publicity. They may consist of different articles, statistical reviews, office records, newspaper records, business records etc. This type of a document could be very useful when the study presumes research on more general and common topics. Personal documents are also very often provided for public access, but the nature of the presented information is completely different. Personal documents may include specific educational statements that are written for a certain focus group, recorded observations of other studies, diaries, portfolios, letters and other such documents. Another important source of information is scientific textbooks with research studies in different areas. This source of information could be related to personal and public documents depending on the provided content. The main advantage of the method is a high availability of data for a study. Obviously, nowadays many topics have been thoroughly studied, and there is plenty of available information with free access. On the other hand, this is a disadvantage as well because due to the high availability of information, some of the data might be inaccurate and incomplete. Hence, the use of such data may lead to misunderstanding and wrong interpretation of the results. The document and literature data collection method is the most popular method and it can be used almost at any stage of research. (Westat 2002, 58)

A case study can also be used as a tool for the collection of data. The case study method is a way of analyzing different cases, situations, processes and model behaviors that have been recorded and measured over time. This method usually provides information and expertise for the development of a certain project in a real-world situation. When implementing the case study method, the researcher is usually searching for similarities and correlations between a studied sample model and the research project, or how the information and expertise acquired in a case study project can be applied to the current research. The case study method might be a useful technique for finding proper data, but it is also a challenging one because very often, it cannot be applied as an illustrative, brief and cursory site visit. On the other hand, the case study method quite often requires a significant time investment and a complex analysis of commonalities between patterns, their possible correlations, and opportunities for development. Usually, the method is initiated by small groups of investigators who study the related documents, hold informal conversations between each other and formal ones with official supervisors as well as discuss results and come to conclusions. The main advantages of the case study data collection method are: it provides a wide picture of the sample situation and shows the possible opportunities and threats as well as the mistakes that have been made previously. The method also provides information about the correlations between the current research project and a previous one through many different perspectives. Since a case study is usually initiated by a group of individuals, it can open the issue from many points of view and bring new knowledge that was previously missed. Nevertheless, the method has its shortcomings as well: the research team has to be well educated and trained for data collection and particularly the case study method because some individual interpretations may lead to misunderstanding of the issue. Moreover, when not initiated by a group of individuals but by one person, the variety of the perspectives and depth of the acquired knowledge might

be lost. The case study method could be best used when the research already has its defined scope, but the proper data has still not been collected or determined. (Westat 2002, 61)

The described data collection methods are the basic and most widely used ones in different types of research. There are plenty of other more complex and diversified methodologies of data collection methods, and some of them have been developed for particularly complicated cases. Each method has its advantages, drawbacks and they must be chosen according to a certain number of research requirements. Furthermore, it is often claimed that successful collection of data for research requires the combinatory usage of different data collection methodologies. The information about the implementation of data collection methods for the current research is thoroughly described in Chapter 2.7.

2.5 Research questions, focus and objective

The scope of this thesis work included two aspects of research: theoretical and practical. The first research question was the primary one and related to the general development of a materials management system:

1) *What inventory control system could best deal with the uncertainty of demand and fit into the operations of Black Bruin Inc?*

Since cost analysis is a very important tool for measuring operational performance, the next two sub questions were added:

2) *What would be the costs of an inventory?*

3) *If it is possible, how could the costs be optimally reduced concerning the constraints?*

The primary aim of this thesis was to develop an inventory replenishment system that could efficiently support the manufacturing operations of Black Bruin Inc. The theoretical objective of the research was to study scientific literature related to inventory and materials management and provide a broad basis of knowledge that would help to clarify the aspects of the studied issue. The practical objective of the thesis was to create efficient technology based on the studied theory that would be utilized in an applied manner and create value for Black Bruin's operations. This means that the technology was aimed to be a real working model especially configured and designed for the case company. The focal point of the technology is to provide a relevant availability level of components for manufacturing while keeping the costs of storing the components low.

The secondary focus of the study was to model and calculate the costs of the inventory. At that point, the primary objective was to determine all the components of the total cost of inventory and identify the key drivers, activities and processes lying beyond them and influencing their magnitude.

The final aim of the research was to seek for the opportunity to reduce the total cost of the inventory through the optimization of decision variables and eliminate extra expenses related to the performance of the inventory operations.

2.6 Limitations of the research

This research was a practice-oriented case study, which was conducted for the company Black Bruin Inc. According to it, all the findings of studied technologies and models were configured and adjusted especially for the

operations and working processes of the case company. Replication of the results, and particularly of the developed inventory system would be difficult, and they would probably be irrelevant to apply even to other similar cases. Further usage or application of the findings to similar cases and problems would require additional research and complementary analyses.

The main area of study was Materials Management, and a general overview of MM is provided in the current study. However, the focus of this research was on studying and implementation of statistical inventory-control models. It means that other areas of materials management, such as Procurement, Demand forecasting, Transportation, Warehousing, Information management are explained only briefly or even not taken into account at all. Moreover, the study explored and analyzed only statistical properties of inventory models (such as stock levels, reorder points, safety stock, etc) and how they were correlated and how they influenced the processes, operations and costs of the company. No physical properties of Inventory and Warehouse management such as the number of racks, warehouse solutions, layout, the design of the work force and other such issues were studied in this research. In addition, sustainability in inventory management was also not covered in this research.

At the beginning of the research process, the author admitted and assumed that the topic of costs' modeling was more related to the field of management accounting and economy, rather than to MM and inventory control. However, the primary aim of this thesis was related to the research in the field of MM and development of an inventory model. Accordingly, the modeling of costs for the particular case were not implemented in an advanced management accounting manner, since this area is not the major field of the author and he simply does not possess deep and profound knowledge concerning this issue. Moreover, based on theoretical knowledge

and experience, the author knew that there was not only one method for costs calculation that could be used in the particular case. According to it, certain components of the costs may vary according to the type of the method used for the calculation. Hence, the modeling of costs in this project cannot be treated as 100% accurate and robust. Nevertheless, the author attempted to create a general picture of cost dependency, its magnitudes and effect on the inventory operations.

The final research question relates to the optimization of the total cost to the minimum objective value. Therefore, the author searched for the opportunities to reduce the total cost of inventory and verify whether the optimization would be relevant with respect to the constraints or whether optimized value total cost of inventory would allow to attain the established performance measures.

2.7 Overview of research structure

According to research focus and objective, the mixed research method was implemented in this study. The method included both of the traditional research methodologies: qualitative and quantitative. Since the nature of the current study assumed the analysis and collection of information represented in two forms, words and numbers, quantitative and qualitative methods were used to a certain extent, as well as different data- analysis and collection techniques.

For answering the first research question, qualitative data collection and analysis methods were more relevant. As the question implies the study and determination of an appropriate inventory control system, the theoretical background of inventory was thoroughly studied. Based on the literature review, traditional qualitative data collection methods were used. The basic method of data collection was studying of scientific literature, articles and other relevant documents related to materials management and inventory

control. The literature review provided the essential information on which the core of the study was based. Secondly, the researcher held informal and formal interviews with Black Bruin's professionals. From those interviews, the researcher collected expert opinions on the theoretical background and verified whether his findings were relevant and how they could be applied to the Black Bruin Inc. case. Furthermore, the researcher explored associated case studies and determined how these models were already implemented and how the empirical experience could be used to eliminate future mistakes. In addition, the researcher periodically worked in the premises of Black Bruin Inc. During work times, he interacted with the company staff and informally observed the logistics processes inside the enterprise. The collected information was also used for the research.

Finally, in order to support the final decision, the chosen inventory model had to be mathematically constructed and validated. At that point, a quantitative data analysis method was used. Particularly, the model was constructed using the Excel spreadsheets and tested with certain input data to identify whether the mathematical mechanism and mechanics of the model were functioning in a correct and an appropriate manner for the BB Inc. case.

Since the second and third questions assume the definition of components of the total cost of the inventory, their mathematical modeling and optimization of the total cost to the minimum objective level, common quantitative as well as qualitative data analysis methods could be carried out for that option. The definition of the components of the total inventory cost were based on the literature review, while the methods for calculation were obtained from scientific literature or interviews with experts. Further, the author mathematically compared various calculation methods and determined which one was the most appropriate for the BB Inc. case. Finally, the author explored several optimization methods and verified which one was the most appropriate for the given case.

Table 2 Research structure.

Chapter	Content
1.	Introduction and company description.
2.	Research methods, design, objectives and limitations.
3.	Theoretical Background and Literature review.
4.	Research implementation.
5.	Discussion and Analysis of Results.
6.	Conclusion.
7.	References.

3. Literature review

3.1 Introduction to Materials management

Materials Management definition.

Materials management is an indispensable business activity, which tends to service and maintain a stable flow of materials or goods (components, raw materials, semi-finished components etc) that are essentially required for manufacturing, logistics or other operations of a company. In particular, materials management is a science that deals with the planning, acquisition, and supply of overall material flows throughout the whole supply chain. (Mishra 2007, 4) According to Mishra (2007, 4), there are four basic types of resources in any industrial or business process: labor, equipment, capital, and material. For a long time, the planning of materials was regarded as a secondary, standard and regular function of an enterprise, and it had not been given proper consideration and studied for many years. However, when the industrial era began its exponential growth, enterprises started to look for opportunities to cut their costs and at the same time to increase their operational performance, and, finally, materials management received due attention and consideration from the authorities. Nowadays, materials management is one of the most important factors that one needs to consider if one wants to have successful, balanced and cost-effective business operations. The area has been studied in many types of research, and today it could be said that materials management is a professional, complex discipline with a focus on many sub-functions, methods, and theories. (Mishra 2007, 4)

The online BusinessDictionary describes materials management as *“The planning and control of the functions supporting the complete cycle (flow) of*

materials, and the associated flow of information. Also, it is called materials planning.” (BusinessDictionary N.D.)

3.1.1 Importance of materials management in the industrial business

Materials management is a complex approach, which combines engineering, economic and management issues at the same time. It tends to explore such activities of the company as outsourcing, global sourcing, purchasing and procurement, production and manufacturing, warehousing and transportation as well as inventory control. It also deals with supplier relationships management, information technology selection and many other relevant aspects. The study of materials management usually deals with such issues as the analysis of purchasing cost, definition of the total cost of ownership and determination of an optimal lot size. Moreover, it also deals with the establishment of relevant inventory levels, safety stock and reorder points as well as the selection of proper transportation or carrier, determination of a suitable supply base and the selection of appropriate information technology for the related operations. (Mishra 2007, 4)

According to Mishra (2007, 5), the role of MM in business activities could be described from three perspectives: as the basic function of a firm, as a manager of outside manufacturing or as a controller of costs.

1. As a basic function of the company, it assumes implementation by the firm of one of the following functions below:

- Design: turning planning into a specification.
- Finance: Controlling the capital and financial flows of the enterprise.
- Personnel: Control and adjustment of Human resources.
- Material: Planning, procurement, transportation and storing of materials, goods and equipment.

- Production: Converting raw materials into final products.
- Sales and Marketing: Distributing products to the end customers.

The implementation of materials management as a function could vary from industry to industry and from company to company, based on one of the functions mentioned above. (Mishra 2007, 5)

- 2. As manager of outside manufacturing:** MM operations are responsible for the management and supply of the parts coming from outside resources. Obviously, an enterprise cannot produce all parts for their final product in their own premises. Therefore, these parts or goods have to be purchased from outer suppliers. One of MM's roles is to maintain these operations. (Mishra 2007, 5)
- 3. As controller of costs:** The third role of MM is related to the control and adjustment of the costs related to supplying, transporting and storing of material. These costs include various logistics costs as well as acquisition production and shortage cost. The aim of MM is to find the balance between the lowest possible cost and the highest operational performance. (Mishra 2007, 5)

3.1.2 Objectives of Materials Management

According to Mishra (2007, 5), MM has three main objectives that could be expressed as:

- 1. Maintaining continuity of supply:** Previously, the stable flow of materials and goods for operations was supported by having high stock levels in the inventory and through the implementation of multiple sourcing procedures. Today, when the technological part of operations has gone through a deeper analysis, it is more likely embodied with help of purchasing portfolios, scrupulous selection of suppliers and development of supplier relationship.

- 2. *Contributing to the reduction of costs.*** This objective is important to study from two perspectives: the first one is the size of the purchase share in the acquired materials, and the second one is the total cost of the ownership perspective or the effect on the bottom line. These could be achieved by using many methods, such as value engineering, purchasing tools, life cycle cost analysis and inventory control models.
- 3. *Innovation into product or process:*** Through a better understanding of procurement's capabilities, innovation might be achieved through the involvement of the supplier at the early stage of product development.

3.1.3 Primary Functions of Materials Management

Many experts view the function of MM from different perspectives and divide them hierarchically in various subdivisions. However, generally speaking, we can state the three primary functions of MM:

1. Material Requirement Planning.

This function includes each activity that relates to capacity and resource planning. MRP is tightly concerned with all manufacturing or production activities, and it determines how much/many raw materials or semi-finished parts are required for operations, how to purchase them when it is sensible for the company and how to make these operations cost effective. For years, MRP disciplines have been developing and obtained a great deal of different methods that could be used for answering these essential questions. One of these methods is using various inventory models, which are aimed at minimizing the total costs of operations. The models could be chosen according to the industry and type of organization. (Mishra 2007, 6)

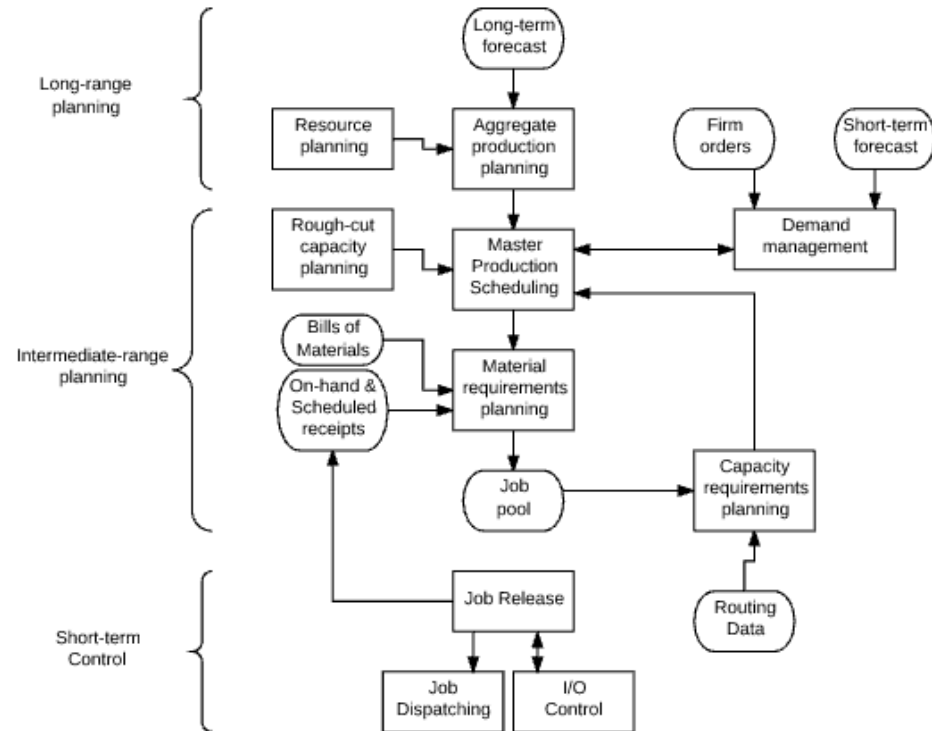


Figure 1. MRP II Hierarchy example obtained from Hopp & Spearman (2008, 140).

2. Procurement.

Procurement, also called industrial buying, is the bottom function of MM. Its primary aim is to support the manufacturing activities of an enterprise by the acquisition of resources that the enterprise is not capable of producing in their premises due to a diversity of the industries or other technical issues. For example, a manufacturer of a mechatronic hydraulic motor system could only produce the components of the hydraulic motor, while the electric components and software are purchased from outer sources. Industrial buying differs greatly from traditional buying in many ways, since materials planning, logistics operations (warehousing and transportation) and cost analysis are much more complicated in this case, and its proper utilization has a serious impact on the operations and performance of an enterprise. (Mishra 2007, 6)

3. Warehousing and Transportation

The final function is mostly related to physical activities and the flow of materials. This property defines such details as which transportation mode to use or which warehousing solution would be appropriate for a particular company case. The selection of the appropriate option in these two factors plays a crucial role in an enterprise's activities as well. For instance, if the selected transportation option is too expensive for the particular type of product, it might have negative effect on the bottom line, and the overall costs on the annual basis may lead to extra expenses and, hence, financial instability of the company. The same could be stated about warehousing or material handling solutions. Since these factors have considerable influence on the overall cost, their management has to be seriously considered.

(Mishra 2007, 6)

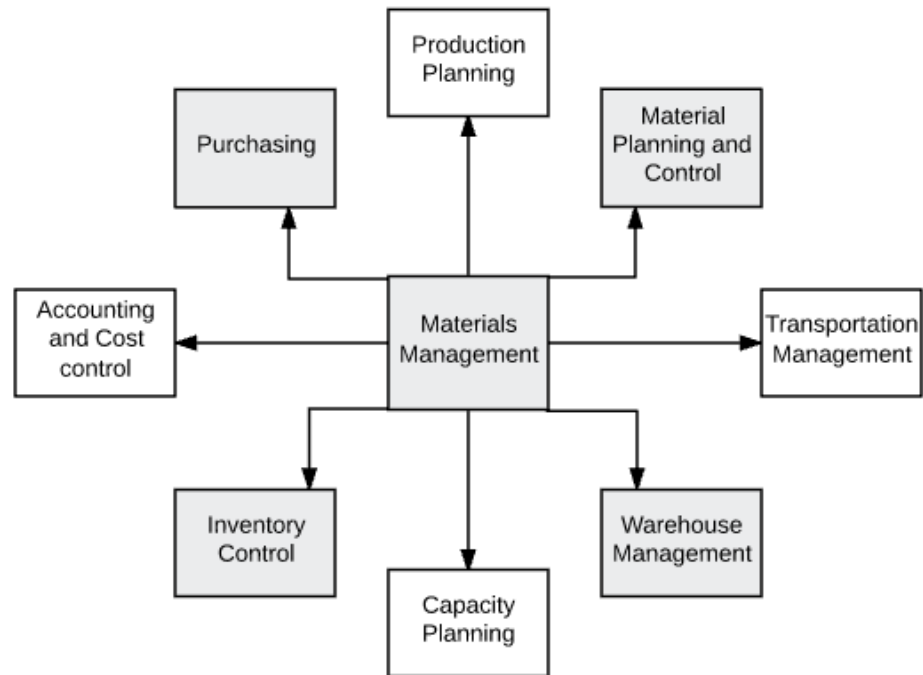


Figure 2. Scope of materials management.

3.2 Inventory

3.2.1 What is Stock and Inventory?

Inventory and stock are essential elements in operation and materials management. In common terms, inventory means a physical presence, or an amount of goods or materials held by an enterprise to be able to support the production process, satisfy customer orders or deal with the uncertainty of demand. Vrat (2014, 21) claims that the Inventory definition from the materials management point of view is: “Idle but usable presence of resources, which has some value held by an enterprise”. At this point, many types of research in operations management area conclude that the concept of inventory is a paradox and should be treated as a “necessary evil”. Obviously, for many companies, it is critically important to have a physical stock and be able to deal with uncertain demand or other external factors,

such as a long lead-time of components. Not considering such principal factors might lead to serious problems, such as production delays and lost sales. Nevertheless, having a physical stock in presence means that an enterprise will have extra expenses, such as “holding” cost, “obsolescence” cost, “energy” costs, “movement” costs etc. Therefore, there is a paradox which states that an enterprise needs to have an inventory, but it is undesirable to have it. According to Vrat (2014, 21), many theories state that an organization should try to reach zero inventory levels, but in reality, it is not possible, since a zero inventory means no buffer against external circumstances. This issue is a perpetual problem area of materials and operations management. (Vrat 2014, 22)

Waters 2003, 4. Defines stock as *“All goods and materials that are stored by an organization. The stock is a store of items that are kept for the future use of an enterprise.”*

Moreover, his definition of Inventory is: *“A list of items that are held in stock.”*

A common problem nowadays is that people tend to mix these definitions, and even many professionals use different terms, for example (Mishra 2007, 6) states that: *“Inventory is the stock of any resource (man, the machine, material) of value kept for the future.”*, while many accountants consider inventory as *“the amount of capital tied up according to some materials or goods in stock.”* (Waters 2003, 4) Obviously, these definitions are easy to confuse, and, therefore, it is sensible to designate them carefully to avoid misunderstanding. For the specification, the current study considers “stock” as it was defined by Waters (2003, 4) and “inventory” term as a standard, but slightly more extended definition:

“Inventory is the list of raw materials, work-in-process products, semi-finished components or finished goods that are considered to be the portion of a business's assets that are ready or will be ready for such operations of the company as manufacturing or sale.” (Investopedia N.D.)

The classification of inventory content could be divided into two definitions as follows:

An item is an individual product or component, which is stored in stock or is a distinct entry that is coming to the inventory. Another definition of this term is SKU or stock keeping unit. An SKU defines the sole product that is distinguished from the other products by common properties, such as color, model, size, brand, weight or even a function. Each item or SKU has its own identification code or number which is recorded in the inventory system and allows to track the amount of SKU present in stock. (Kisler 2014) When the item or SKU is sold or utilized in the operations of the company, the quantity of the utilized items is defined as a unit. According to that, a unit is a standardized number of items held in inventory for the support of industrial processes (Waters 2003, 4-7)

3.2.2 The reasons for holding inventory

Why do we need inventory?

As it was mentioned before, in an ideal situation the enterprise should try to aim at zero inventory levels. However, zero inventory makes no sense, as then an organization will not be able to buffer against shortages, delays in the production and other external factors. According to Vrat (2014, 23) the reasons, why inventory is needed in industrial business could be classified as follows:

1. The amount of time between the point when an order is placed and between the point when the order arrives in a warehouse which is called lead time – whenever the order is placed, there is a time gap before this order will arrive. Commonly, in operations management lead time refers to a difference between the start of the certain process and an end of this process. (Bharath & Prakash 2014, 35) In materials management, the term lead-time could relate to many activities, however, in the current study the definition will mostly relate to the time gap between the placement of order and its arrival. Consequently, in the majority of cases, lead-time of supply has non-zero value and very often it takes significantly longer period to supply the components from one organization to another. According to it, inventory allows taking care of the demand during lead times. (Vrat 2014, 23)

2. Another factor that influence the amount of inventory is the variability of lead-time. In many cases, the particularity of the industry or supply chain presumes that there is lead-time variability to a certain extent. Very often, the situation is so that standard deviation of lead-time from the mean might be around three to four weeks, while the mean value is ten. The numbers are critical for resource management since an overall range of uncertainty is from six to fourteen weeks. This situation is very risky as it increases the probability of the stock out significantly and traditionally a supplied organization cannot affect them in another way than holding some amount of inventory. Therefore, inventory is a “shield” which buffer against this uncertainty. In addition, the amount of holding the stock is directly depended on the variability in lead-time. The higher suspense in a supply period – the higher levels of inventory an enterprise should maintain. (Vrat 2014, 23)

3. Variability in demand – probably is one of the main factors why organizations create inventories. According to Kärkkänen (2015) the inaccuracies in demand forecasting are a common phenomenon, and in the

majority of industries, it is almost impossible to provide a fully accurate forecast. Furthermore, Gilliland (2011) claims that many types of demand are unforecastable and the only values you can get are approximate figures of the real situation. The ambiguity of this point is crucial and has to be protected somehow. One of the ways to do it is to have stock that would provide an ability to withstand against it. Moreover, the variability in demand directly influences the amount of required inventory. (Vrat 2014, 23)

4. Sometimes the reason for inventory accumulation could be caused by seasonal or cyclic demand. In case if the peak period of demand falls on some particular period and production does not have enough capacity to fulfill it right at the time when it occurs, building inventory in a lean period could be a solution for that case. (Vrat 2014, 23)

5. The fact of product or commodity distribution over long distances could also be one reason for inventory accumulation. This is done to provide substantially important availability of goods, which are “in transit.” The procedure is called pipeline inventory. Sometimes pipeline inventory is established in production line during manufacturing processes and is called WIP or work in process inventory. (Vrat 2014, 23)

6. Inflationary pressures, shortage of materials in the markets, and bulk purchases of commodities due to seasonal discounts or economies of scale are also the reason for inventory accumulation. These factors are related to inessential needs of having an inventory, and very often they result in dead inventories. (Vrat 2014, 23)

3.2.3 The role of inventory

Nevertheless, despite the fact that inventory is claimed as ‘necessary evil’ it is also very important part of operations management and implementation of proper inventory control might be very beneficial for its owner. Correctly implemented inventory control techniques will provide clear transaction of

capital tied up to stocks, improves the responsiveness of the enterprise to external fluctuations, increase customer service and eliminate the possible difference between accounting and real inventory levels. (Kiisler 2014)

The role of inventory in operations management is also inevitable since it influences many activities of an enterprise differently. For example, production planning is completely dependent on inventory control and replenishment system, purchasing policies have to take into consideration inventory control features, planning of warehouse space almost always depends on inventory system that is used by an enterprise. (Waters 2003, 10)

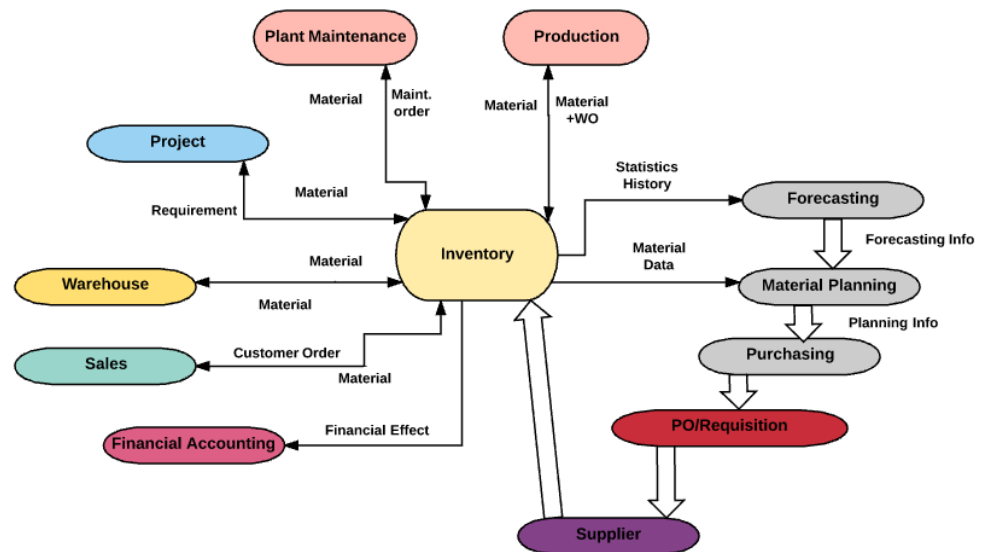


Figure 3 The role of inventory in Operations Management.

3.2.4 Inventory strategies

According to van Heck (2009, 42-44) inventories are the sources of risk for the enterprise, they require a significant amount of money from working capital and need some resources to maintain them. Contrariwise, in many cases, it is impossible to operate without having some amount of stock on

hand. As the previous chapter has shown, inventory is an indispensable part in the operational management, and most probably, the industry will never be able to eliminate them. Due to that fact, the most important question is: what is the optimal amount of stock to have on hand? Univocal answer on this question does not exist. The amount of inventory, which organization needs to have on hand depends on many different factors and has to be viewed individually, depending on the capabilities and operational features of the organization. (van Heck 2009, 42-44)

Corporate strategy.

One of the most important factors, which influences on the amount of inventory on-hand is a corporate strategy. Corporate strategy is usually established by management. Based on this, the decisions that are taken about inventory operations have to take into consideration aims that are set by corporate strategy. The setting of service level value is an example of corporate strategy decision that will affect inventory operations. (van Heck 2009, 42-44)

According to Treacy (1993, 84-93) and Dijk (2007, 99) The three-main key corporate strategies that are achieved by inventory management are:

1. Operational excellence means to offer high-quality products to a customer with lowest possible prices.
2. Customer Intimacy; this strategy means that an enterprise should aim to build very close relationships with their customers through adjustment of products and services according to individual needs of every customer. As an example, high service level values, perfect order fulfillment, short lead times could be very important goals or sub-goals of this strategy.
3. Product leadership is another example of the corporate strategy. When implementing product leadership, the organization should aim at

innovation that will distinguish their services and products from the competitors. Apple could be an example of product leadership strategy. (Treacy 1993, 84-93) and (Dijk 2007, 99)

According to Dijk (2007, 99), an example of corporate strategy implementation could be good demonstrated on the grocery stores Lidl and Albert Heijn. Lidl's corporate strategy is corresponding to the operational excellence pattern, a lot of emphases is put on the product price and its quality. However, the image, interior at Lidl is relatively poor comparing to Albert Heijn. Statistically, usually Lidl does not provide any additional discounts rather than drops in prices for products which expiry date is close and what is more important, product variability is small, while the stock out situation at Lidl is more likely to occur. On the other hand, Albert Heijn is implementing the corporate strategy regarding customer intimacy, and the emphasis of this brand is to provide excellent customer service with relatively higher costs comparing to Lidl. The products' variability, the interior is much better in Albert Heijn, while the probability of stock out is much less likely to happen. In addition, Albert Heijn has specific customer relationship system, which provides special discount for permanent customers. At this point, it is relevant to consider the inventory management at both of the grocery stores and overall logistics processes regarding their difference according to the corporate strategy. (Wild 2002)

Business Model.

According to van Heck (2009, 42-44), another very important factor is a business model that should be taken into account when choosing inventory strategy. Business model means a manner at which an enterprise is implementing its operations, also called a basic structure. Hoekstra & Romme (1991) and Dijk (2007, 38) defines four basic structures as:

1. Make-to-stock. According to this business model products are manufactured in anticipation of product demand and then stored in inventory until the order arrives. Sometimes it is even possible that products are manufactured at a period when there is no demand at all. In that case, a lot of pressure is released on sales department that is supposed to implement distribution operations.
2. Deliver-from-stock. This option has a lot of similarities with previous one Make-to-stock business model. However, the difference is that variety of product is much wider, and the products are stored not in the premises of the manufacturer, but in the premises of wholesaler or retailer.
3. Assemble-to-order. The principle of this model is based on the fact, that some manufacturing companies only assemble final product, while manufacturing of components is implemented at premises of suppliers. The method is initiated when final assembler holds some limited number of components at his inventory and assemble them into final product when the order is received. Customization often takes place at this business model. It allows the company to adjust product according to an individual customer needs.
4. Make-to-order. This business model is very similar to the previous one mentioned. Nevertheless, some of the components here are produced from raw materials, and then it goes along with other purchased components into finished product. The method is initiated according to the customer demand (so when an order arrives). Customization is a very important aspect since make-to-stock often assumes working with individual customers and satisfaction of specific customer's needs.

These basic structures are not the only ones, and some variations exist. For example, according to Hoekstra & Romme (1991), there also exists fifth structure which is defined as "purchase and make to order."

The basic structure or business model is a very crucial factor when determining an amount of stock to hold. For instance, in make-to-order model components cannot be just stored in inventory, since they are required only according to customer demand. Nevertheless, if the lead-time of components is high, some amount of material is necessary to keep in stock to be able to satisfy customer orders. In such situation, it is very important to determine the proper number of components to store. If the stock would be too low, there is a possibility of stock out and lost sales. Otherwise, if the stock is too high, there is a possibility of overstock that will lead to extra expenses. (van Heck 2009, 42-44)

Different views on inventory.

Consideration of inventory from different perspectives also plays a very important role when taking into account inventory levels. For sales department, the service level is a very important factor, as they aim to satisfy as high as the possible amount of customers' orders. However, for the management, the service level is just one of the factors that have to be in the balance with other ones. Very often, the aim of the management is to reduce costs and through that improve operational performance. While keeping the high level of stock for the service level means that a lot of working capital and other expenses will be required. The misbalance between these two points is a stumbling-stone of inventory management. (van Heck 2009, 42-44)

In addition, an example of different perspectives could be the situation, when procurement manager has to purchase goods, and the price of them is crucial for him. If there is a possibility to purchase a huge batch of components at once (economy of scale), it will seem a beneficial option. However, for the inventory manager, it means maintaining a high level of stock that will lead to extra costs and higher working capital. The tension

between different perspectives is a perpetual problem of inventory management and the matter of the question “what is the optimal amount of stock to have on hand?” (van Heck 2009, 42-44)

3.2.5 Lean Inventory Management

The idea of the Lean concept is associated with Kiichiro Toyoda, Taiichi Ohno, and Japanese auto manufacturer Toyota. The essence of lean is referred to all activities across the supply chain and their enhancement to achieve higher operational performance through the reduction of costs and elimination of all forms of waste. The first emergence of Lean concept was noticed at 1930 when the Kiichiro Toyoda and Taiichi Ohno visited the production facility of American auto manufacturer Ford, which was based on the continuous-flow manufacturing and defined the seven waste forms that occur in the manufacturing and related activities. According to the Goetsch, & Davis (2010, 538-539) the seven wastes are arising from the seven sources respectively: 1) Overproduction 2) Excessive waiting time 3) Over transportation 4) Over processing 5) Excessive stock on hand 6) Unnecessary motion 7) Waste coming from producing defective goods. (Goetsch & Davis 2010, 538-539)

Nevertheless, despite the fact that Toyota members have made a great contribution towards Lean Concept, its original appearance on the scene was 1990 in the book *The Machine That Changed the World: The Story of Lean Production* by James Womack, Daniel T. Jones, and Daniel Roos.¹ (Goetsch & Davis 2010, 538-539)

Taking into account Lean Inventory Management, it associates with a methodical manner of inventory control that seeks to increase efficiency and value of company's operation through a continuous elimination of waste in material planning, establishment of proper inventory levels and permanent process' improvement. (Sheldon 2006, 35-39)

According to Sheldon (2006, 35-39), Lean Inventory strategy is often accounted as an approach to reduction in inventory levels. However, the Lean stock strategy does mean the reduction in stock. The main properties of Lean concept such as flexibility and responsiveness do not tend to deny having buffer inventory on hand, on the other hand, even Toyota Production System that tightly connected to the concept of Lean admit the presence of necessary stock buffers if it is required. To provide better understanding of the Lean inventory, it is necessary to understand the reasons for inventory existing and the purpose of lean strategy. Sheldon (2006, 35-39) defines the first one as a 'strategic weapon' that supports stable material flow for manufacturing, while second is defined as the way of continuous improvement in operational performance through the permanent enhancement of activities and management of enterprise's resources. Based on these definitions, the concept of Lean inventory represents the systematic approach in inventory control, that aims to provide required stock levels when it is needed. Accordingly, Lean strategy in inventory management does not mean the reduction in stock levels for the sake of reduction in stock levels but aims for its optimization through the permanent development of activities that reduce the amount of required inventory buffer. (Sheldon 2006, 35-39)

3.2.6 Just-In-Time Inventory

The just-in-time concept is a Japanese management philosophy that was firstly applied to manufacturing systems. Nowadays, JIT is widely known management approach that is broadly used in operations management. One may ask "What does Just-in-Time stand for?". Initially, the answer could relate to the essence of manufacturing philosophy meaning "producing only what is needed, when it is needed, and in the needed quantity." (Goetsch & Davis 2010, 538-539) However, when it comes to inventory control, the more appropriate answer would be "JIT is an approach that aimed at

matching of supply and demand in a way that needs for inventory could be eliminated.” (Waters 2003, 342-343) Nevertheless, coming backward to the number of core reasons for inventory keeping, one may get confused with the contradiction of JIT philosophy and its’ application to inventory systems, since both concepts intersect with each other in the absolutely opposite directions. To obtain a better understanding of the issue, it is necessary to mention that JIT philosophy’s core principles were literally developed by the same people, who have decried the seven wastes in Ford’s continuous-flow manufacturing system, Japanese Engineers and Toyota Production System pioneers: Kiichiro Toyoda, Taiichi Ohno. At this point it is wise to refer to the previously mentioned Lean Philosophy, as it is tightly connected to the JIT and they both contribute to the elimination of all forms of waste, rather than Lean concept is much wider approach of decreasing an amount of waste, while JIT denotes more particular manner in the waste’s reduction. Even so, the primary purpose of JIT philosophy is to eliminate all forms of waste through the provision of required material only when they are needed, in that sense JIT sees stocks as unnecessary resources that are not performing any effective function rather than eating money and waiting to be utilized. This statement completely contradicts the conventional view of the inventory, which refers stock as a buffer against external uncertainty. In this perspective, the general question of inventory control “How much stock to keep?” is changing to the new one “How is it possible to eliminate the need for the stocks?”. The essence of the answer provided by JIT philosophy is saying “define the root cause of the need for stock and resolve it.” In other words, the potential reasons that motivate an organization to acquire inventories must be resolved. For instance, if the motivation for holding the stock is demand’s uncertainty, the solution is to dispose of the uncertainty by removing the cause which has evoked it. Or if the reason for inventory keeping is disruptions in supply, JIT says these disruptions must be eliminated. All in all, despite the fact that JIT seems to be a simple idea at a

glimpse, in fact, it needs significant changes in the management of an organization. JIT inventory is an intelligent manner to handle stock control, but its' implementation is not possible in certain business sectors due to the particularity of supply chains or the specificity of the industry. (Waters 2003, 343-345)

3.2.7 Inventory components

Cycle stock.

Whenever an enterprise acquires items or materials from outer sources, it is obvious that they are not utilized promptly. On the contrary, the acquired materials are in many cases stored by the organization for future use. The general methodology is that items are purchased and delivered to a warehouse in an optimal size. Afterwards, they are consumed until the inventory level reaches a certain point. This value is statistically determined and called the reorder point. When this point is reached, the organization places a replenishment order with a certain amount of material and waits until this order arrives. (Waters 2003, 4-7) An important consideration is that the consumption of items stored in the inventory during the lead-time remains constant in basic cases. In other words, since the inventory is replenished with a certain unit of items, the enterprise starts to consume them for operations or sales, and when the item's level reaches a certain point, the order is placed again, while during the lead-time of the order, the utilization of the inventory is continuing. This replenishment sequence is represented or defined as a cycle stock. According to (Waters 2003, 4-7), cycle stock is another statistically determined dynamic value, which is one component of a statistical inventory and it could be described with certain properties:

- Cycle stock is a dynamic value that is always decreasing in an ideal situation
 - Cycle stock represents the average amount of inventory units consumed by an enterprise between the point of placement and arrival of a replenishment order
 - Cycle stock has a reorder point, which determines the time when a new replenishment order has to be placed
- (Waters 2003, 6-7)

The graphical representation of a cycle stock can be seen in Figure 4.

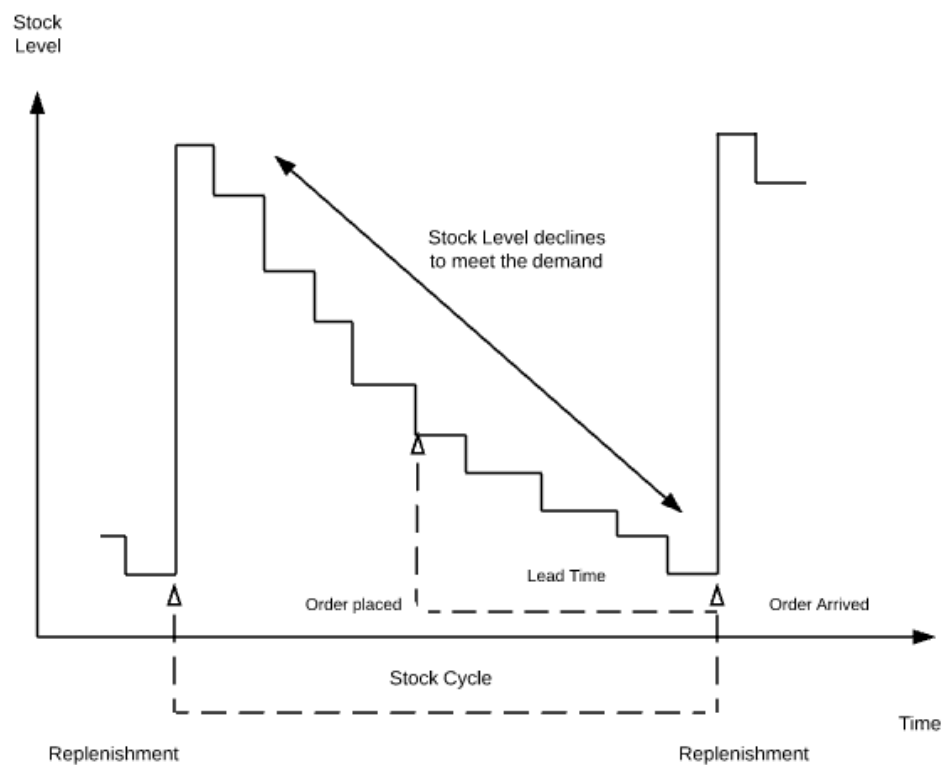


Figure 4 Cycle stock graph (Adapted from Waters 2003, 6-7).

In addition to that, cycle stock value could also be defined analytically, using the following mathematical formula:

$$TACS = \sum_{i=1}^n \sqrt{\frac{A * D_i * v_i}{2r}} = \sqrt{\frac{A}{r}} \times \frac{1}{\sqrt{2}} \sum_{i=1}^n \sqrt{D_i * v_i} \quad (1)$$

(Caplice 2006)

Where: TACS = Average cycle stock, A = order common cost for all items, r = carrying cost for all items, D_i = Demand for item I, v_i = Purchase price for item I. (Caplice 2006) In the chapter 3.3.3 of this paper more customary representation of this formula is presented as Economical Order Quantity formula. The essence of EOQ is thoroughly explained in the chapter 3.3.3.

Safety stock.

As it was mentioned in the previous section, the cycle stock describes an average number of items in inventory during actual demand and demand during the lead time of materials. However, what one is supposed to do if the number of items in inventory will reach zero points, while order has not yet arrived? Unfortunately, this situation means stock out and in the majority of cases; it is one of the most frightening nightmares for operations' managers since stock out usually means lost sales and sometimes even lost customers. (King 2011, 33-35) According to the statement, it is useful to establish some sort of buffer that will protect an organization against those situations. This buffer is a safety stock. The idea of safety stock is to have a

certain number of items and use them only in critical situations when stock out has occurred. Stock out usually happens because of such factors as fluctuations in customer demand, variability in lead times of manufacturing/other logistics activities, inaccuracy in the forecast or other external factors that could not be known in advance. Nevertheless, safety stock is not aimed to eliminate all stock outs – there is still fracture of probability that stock out might occur and this fracture is determined by service level factor. Service level is a percentage value, which shows the fracture of replenishment cycles where stock out could be expected. (King 2011, 33-35) Figure 5 represents this dependency graphically.

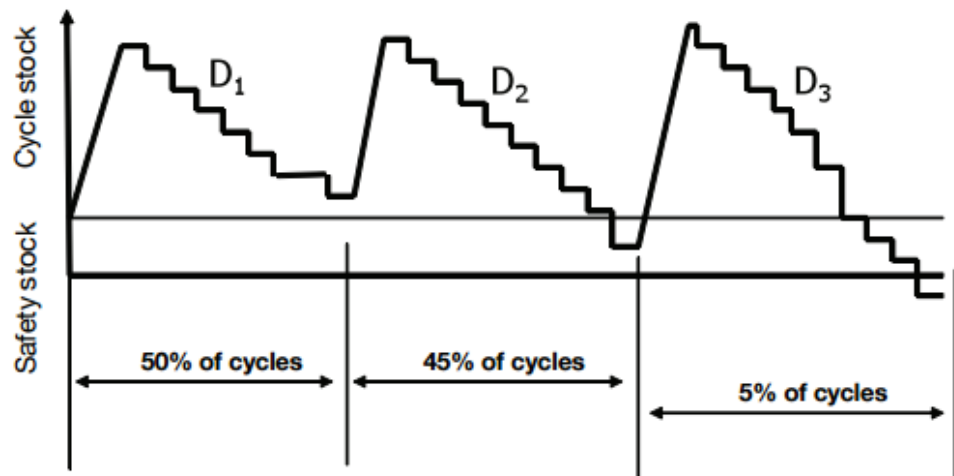


Figure 5 Safety stock vs Service level (King 2011, 33-35)

According to the Figure 5, it can be seen that during 50% of the time not even all amount of cycle stock would be consumed. While during 45% of the time some portion of safety stock would be beneficial to utilize, but during

the rest 5% of the time, the stock out situation still could be expected. (King 2011, 33-35)

According to Waters (2003, 171), an alternative definition of service level is the objective proportion of demand that is filled directly from stock or, on the other hand, the service level is maximum probability accepted by an organization that they will not be able to fill the demand from stock.

King (2011, 33-35) states that in common practice, the percentage value of SS is a statistical representation of the number of situations where organization aim to satisfy customer demand and avoid stock out. According to that, the value of service level and the amount of SS are directly dependent on each other. In other words, for the service level of 97%, an enterprise must hold more SS than for service level of 95%.

The service level factor is determined by Z-score or service factor. Z-score is a coefficient that we get from the normal curve, and it represents some standard deviations away from the mean. (Lähdeväärä 2014) According to Radasanu (2016, 149), the control between service level and safety stock could be implemented through the use of standard deviation. In detail, normal distribution determines the data range where the majority of values are situated close to the mean value. Simultaneously, a small number of values is situated at extreme distances from the mean value, while a total number of values existing below mean value is close to the total number of values existing above mean value. According to the Figure 5, in accordance with normal distribution, 99.73% of data is situated in the range of three standard deviations of the mean value ($x \pm 3\sigma$), 95.45% of data is situated in the range of two standard deviations of the mean value ($x \pm 2\sigma$), while 68.26% of data is situated in the range of one standard deviation of the mean value ($x \pm \sigma$). (Radasanu 2016, 149)

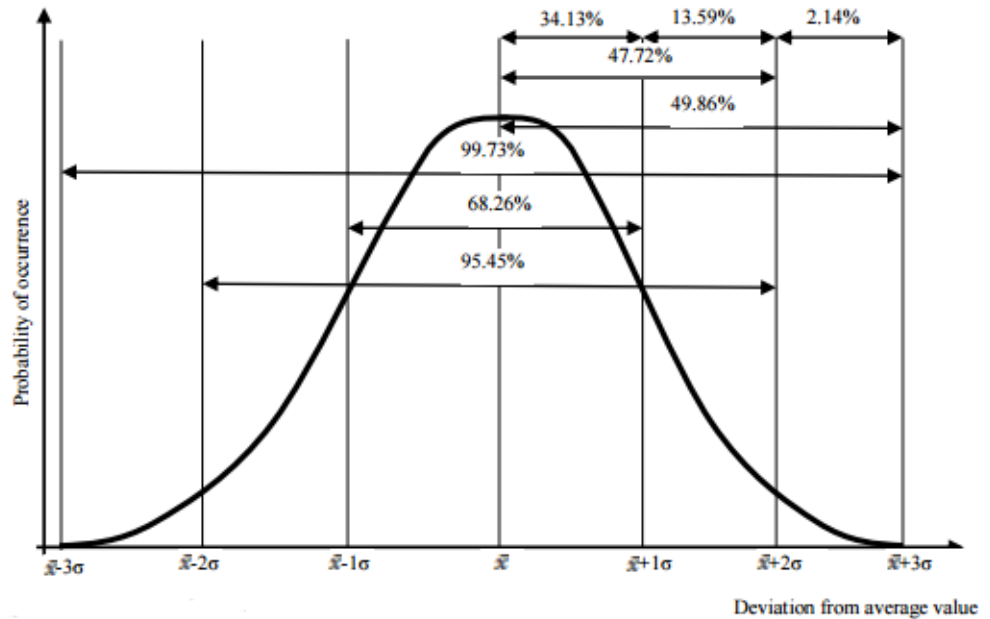


Figure 6 Normal distribution according to Gauss (Hoppe 2008, 373)

In this case, the Z-score is used to calculate particular quantity for meeting specific service level through its' multiplication with the standard deviation. (Radasanu 2016, 149)

The relationship between desired service level and Z-score is represented in Table 3.

Table 3. Correlation between desired service level and Z-Score

Ser. Level	Z-score	Ser. Level	Z-score	Ser. Level	Z-score
99.99%	3.719	97%	1.881	85%	1.036
99.8%	2.878	96%	1.751	80%	0.842
99.7%	2.748	95%	1.645	75%	0.674
99.6%	2.652	94%	1.555	70%	0.524
99.5%	2.576	93%	1.476	65%	0.385
99%	2.326	92%	1.405	60%	0.253
98.5%	2.170	91%	1.341	55%	0.126
98%	2.054	90%	1.282	50%	0.000

Safety stock value could be found analytically according to mathematical formulas.

Depending on the case, whether variability in demand, variability in lead time or both are present, different formulas could be used.

According to demand variability:

$$Safety\ Stock = Z \times \sqrt{\frac{PC}{T_i}} \times \sigma_d \quad (2)$$

(King 2011, 33-35)

Where: Z = Z-score, PC = performance cycle or Lead time, T_i = is time increment used for calculation of standard deviation in demand, σ_d = standard deviation of demand.

According to Lead time variability:

$$Safety\ Stock = Z \times \sigma_{lt} \times D_{av} \quad (3)$$

(King 2011, 33-35)

Where: Z = Z-score; σ_{lt} = standard deviation of lead time; D_{av} = Average demand

According to variability in Lead time and Demand:

When they are dependent:

$$Safety\ Stock = Z \times \sqrt{\left(\frac{PC}{T_i} \times \sigma_d^2\right) + (\sigma_{LT} \times D_{av})^2} \quad (4)$$

When they are independent:

$$Safety\ Stock = \left(Z \times \sqrt{\frac{PC}{T_i} \times \sigma_d} \right) + (Z \times \sigma_{lt} \times D_{av}) \quad (5)$$

(King 2011, 33-35)

Surplus stock.

Surplus stock is an undesirable excessive inventory, which usually produced due to miscalculation concerning actually required inventory levels. There are numerous reasons why surplus stock can occur: severe decreases in demand, miscalculation of factors influencing safety stock, wrong forecasts, wrong estimation of the required service level, over-purchasing of materials due to an economy of scale, inadequate material requirement planning to mention but a few. Surplus stock is an incidental component of inventory, and it does not create any value for an organization. Accordingly, one of the primary aims of the inventory manager is to eliminate the presence of excessive stock. (Willoughby, 5-6)

According to Silver, Pyke and Peterson (1998), one way to estimate surplus stock is to use the item-by-item rule. This methodology computes the

expected time for the stock to be consumed. The value is defined as the item's coverage and its formula:

$$CO = (12 \times I)/D \quad (6)$$

Where: CO = coverage time (months); I = inventory on-hand; D = expected demand (units/year).

The estimation of excessive stock is important since it could help to provide operation managers with the facts of the surplus stock significance. Besides this, it could also help to indicate the costs of excessive inventory and consequently eliminate extra expenses. (Willoughby, 5-6)

3.2.8 Costs of Inventory

According to Vermorel (2013), the inventory costs are the costs of acquiring, holding and maintaining a certain number of items in the inventory for a particular period. Typically, the costs of inventory are 'static' costs that are related to the process of receiving an item, storing it and to the ability to respond to customer orders. General practice in inventory accounting is that these costs are distinguished from those related to movements in the warehouse and flow of materials. Particularly, the inventory costs are focused only on the expenses related to the ownership of a distinct inventory value. (Vermorel 2013)

Challenges in Estimation of Inventory costs.

The estimation of inventory costs is a challenging and complicated procedure. The reason for that is that real inventory costs go far beyond the simple value

of the products that are stored in a warehouse. (Vermorel 2013) According to Silver, Pyke and Peterson (1998), depending on the industry type, the different algorithms may produce relatively precise answers in one case and could be absolutely irrelevant for another case. The crucial variability is motivated by differences in the type of industry, type of products or materials stored, specificity of supply chains and, simply, the accounting systems or estimation methods that are applied by an organization. For example, Vermorel (2013) states that a very important factor is whether the costs are calculated after taxation or before it, and the combination of after-tax and before-tax methods is not allowed since it will result in absolutely wrong figures. In order to gain a better understanding of the drivers that hide behind the inventory costs, it is sensible firstly to define the inventory costs.

Defining inventory costs

According to Hopp & Spearman (2008, 66-67), the inventory costs are divided into four categories: Ordering cost (Setup cost), Holding cost, Backorder cost and Stock out cost.

Ordering cost or **setup cost** is a cost related to the process of placement a replenishment order and receiving it in a warehouse. According to Vermorel (2013), the main drivers behind the ordering cost are generally two processes: the cost of ordering process and the process of inbound logistics. The ordering process is usually accounted for as a fixed cost since it does not vary with the number of units ordered and consist of fees related to clerical activities, such as sending an invoice, communication between the supplier organization and the customer, printing the documents related to the order etc. The inbound logistics process is, on the other hand, a variable cost and it may differ depending on how much material or goods are ordered. This process usually consists of activities related to transportation, inspecting the cargo, unloading it in a warehouse and other such operations. This type of cost is difficult to estimate since the evaluation of such activities, such as sending an invoice is

challenging and, frankly speaking, almost no accountant can say how much it will accurately cost. (Vermorel 2013)

Holding cost according to the Vermorel (2013) tend to be significantly underestimated by the majority of organizations, and it may reach up to 25% of inventory value depending on the organization. The holding cost is the biggest cost of inventory, and it primarily consists of four components: Capital cost, facility cost, inventory service cost and inventory risk cost.

Capital cost is the largest element of holding cost and according to Vermorel (2013) mainly includes all factors related to inventory investment, working capital interest rate, the opportunity cost of invested money, etc. The capital cost may reach up to 15% of the inventory holding cost, and its estimation is rather sophisticated and complex process. One of the possible solutions is proposed by Timme S.G and Timme, C.W. (2003) to calculate capital cost through the use of WACC (weighted average cost of capital). WACC is representing the rate a company is expected to pay on average to all its security holders to finance its assets (Vermorel 2013) The solution of Timme S.G and Timme, C.W. (2003) for WACC calculation is expressed through the costs of equity and aftertax cost of debt. The equation for WACC will be derived later in this paper. *Facility costs* are related to expenses that needed to maintain the functionality of warehouse facility: heating, lighting, air conditioning, electricity. This type of costs is extremely dependent on the type of warehouse facility, warehouse solution or a number of racks. For example, warehouse facility with automated racks will result in a bigger interest rate for facility costs, since automated equipment value is higher and its' maintenance consequently require more resources. The presence of refrigerators or other specific equipment that is needed for storing of materials may affect the resultative cost as well. Another factor which may influence this is cost is capacity of the warehouse, if there is some extra capacity which is not utilized, it could result in higher facility costs per unit of

inventory. Inventory service costs are the class of costs related to inventory management or inventory control. The elements of this cost type could include insurances, expenses associated with IT management system, some tracking hardware as RFID, etc. This category could also include the outcomes related to administration, human labor related to physical handling and management on inventory system. The last classification of holding cost is Inventory risk cost. This cost is associated with the expenses that could occur if the items would be lost, stolen, obsolete, damaged or if they would simply lose some value during the time of storing. Costs related to risk are very sensitive when it comes to dairy products, perishable products, or sometimes even to the consumer electronic components that may obsolete during the storage time and lose their value. Vermorel (2013) States that estimation of inventory risk cost is not as direct as it may appear at a glimpse. On the other hand, an establishment of the proper risk interest rate is considered to be a complicated task, since very often this number is based on prediction and it cannot account for each motivating element that is lying beyond it. According to Vermorel (2013) when defining the factor of risk, it is sensible to consider the value of write-off over a given period. Nevertheless, write-offs are often determined in a wrong way, and resultative risk factors may be inadequate. (Vermorel 2013)

Backorder cost or Stock out cost is the type of cost which is used to penalize poor customer service. Backorder cost is an approach to charge poor service whenever the customer order is not met by postponing it until there will be a sufficient amount of goods in inventory to respond to it. Backorder cost then is proportional to the time customer has to wait until the order is completed by an organization. Stock out cost, on the other hand, is a manner to charge poor service whenever order cannot be filled from stock by charging penalty cost. Regrettably, these types of costs are the most complicated to estimate since it includes intangible parameters that

represent the loss of customer goodwill and the loss of company reputation (Hopp & Spearman 2008, 83) Empirically it was seen that these parameters almost impossible accurately define in reality, however, their existence is a fact, and its' value may play a crucial role when it comes to the inventory control systems. Modelling of these types of costs is an absolutely distinctive and challenging task that requires separate research, and it goes far behind the calculation of conventional inventory related expenses. (Vermorel 2013)

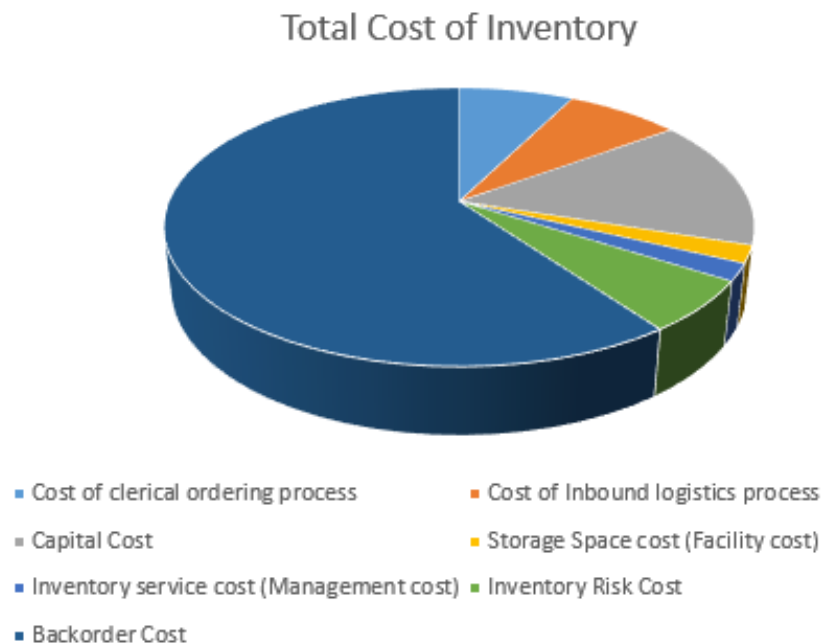


Figure 7. Total Cost of Inventory distribution according to Vermorel (2013) visualized by the author.

Methods for calculating the costs of inventory

There is a number of methods to calculate the costs of inventory, and they vary due to the type of specificity of business and accounting system used in the organization. (Vermorel 2013) According to Sipilä (2017) and Koiranen (2017) one manner to calculate the inventory related expenses is to established interest rate factor expressed in percentage from the value of items that are stored in inventory. For instance, if the item value is 100€ and

the interest rate for the cost of the facility, management cost and cost of risk are considered to be 1%, 2%, and 3% respectively, then the entire cost is computed as:

$$100\text{€} * 1\% + 100 * 2\% + 100\text{€} * 3\% = 1\text{€} + 2\text{€} + 3\text{€} = 6\text{€} \quad (7)$$

The challenging point in this approach is to find proper interest rate and motivate the factors that are hiding beyond.

Almost the same approach could be applied to the cost of clerical ordering process and cost of inbound logistics process. However, these costing components usually are not influenced by the value of an item, since the ordering process implies implementation of the same activities and the value of the item does not affect it. In case of cost for ordering process, it is more relevant to identify the value for each component of the ordering cost. This value could remain the same in case if the item is ordered from the same supplier and essence of the ordering process is identical. (Sipilä & Lehtola, 2017)

Nonetheless, other elements related to the inventory holding cost could be classified as non-capital inventory costs, and basically, interest rate method could be applied to almost all non-capital costing components. To establish as much as possible accurate interest rate itself is a very complicated and challenging task. If one will consider this procedure with thorough approach, it is wise to implement separate research in the field of inventory accounting and define all the possible cost drivers lying behind these percentage values. (Sipilä; Koiranen 2017)

However, when it comes to inventory capital costs, the interest rate approach is not wise to implement since capital costs represent sophisticated financial flows related to inventory investment, working capital

interest rate, the opportunity cost of invested money, etc. and interest rate method simply cannot properly account all of them (Vermorel 2013)

According to Timme S.G and Timme, C.W. (2003) the one method to calculate capital cost is to use the WACC (weighted average cost of capital) method. As it was already mentioned before, WACC is representing the rate a company is expected to pay on average to all its security holders to finance its assets and usually associated with the firm's cost of capital. Simplistically Nuno (2014, 32) defines the WACC as the minimum return of money that organization must earn on current assets to be able to gratify its stakeholders, creditors, owners and other capital holders to prevent them to invest money elsewhere. The inventory is one example of assets that are considered to have this money return. According to Timme S.G and Timme, C.W. (2003) the WACC calculation could be computed through the formula:

$$WACC = E \times CE + D \times CD \times (100\% - MTR) \quad (8)$$

Where: E = Equity, CE = Cost of Equity, D = Debt, CD = Cost of Debt and, MTR = Marginal Tax Rate.

Important to notice that due to the formula provided Timme S.G and Timme, C.W. (2003) it is necessary to specify how the cost of equity is expressed: based on the after-tax value or before-tax value.

The last classification of costs related to the most challenging costs to determine: Backorder cost and Stock out cost. This category is the most challenging one to define since it represents the loss of customer goodwill and company reputation. In addition, the essence of this category is based on intangible measures such as a proportion of time customer has to wait before an order is fulfilled (backorder case) or the proportion of lost customer goodwill to the number of orders that cannot be fulfilled from

inventory (stock out case). Unfortunately, the specification of these cost-type requires additional research, which is not sensible to implement in many SME due to the number of reasons. (Hopp & Spearman 2008, 83-84) However, as it will be seen later in this research these costs are playing important role in some types of inventory models, and they have to be set somehow. According to Hopp & Spearman (2008, 83-84), one way to define them is simply estimate and see how they will affect such output performance measures as Fill rate and Backorder level. If the calculation will produce values that are far from reality (for example fill rate 50%), it is wise to adjust the them until the output result will more or less correspond to the realistic numbers. Afterwards, it is also advisable to implement sensitivity analysis of these costs and define the range where the values tend to be objective. (Hopp & Spearman 2008, 83-84)

3.2.9 Types of Inventories based on the demand pattern

There are many factors on the basis of which inventory levels could be computed. One of these factors is a pattern of demand. Inventory control techniques could be classified according to the type of demand. Primarily, there are two types of demand for inventory: Independent demand and Dependent demand. (“MSG Management Study Guide.”)

Independent Demand Inventories.

According to Mentzer and Moon (2006), independent demand is the amount of product demanded (by time and location) by the end use customer of the supply chain. Depending on the factor, whether the end user is an individual consumer at the shopping center (B2C case) or business partner at the end of upside part in a supply chain (B2B case), these types of end user will define the final actual demand of the product in a supply chain. “MSG

Management Study Guide.” states that an inventory of an item is classified as independent if the item refers to the class of independent demand or when the demand for this item is not dependent on the demand for another item. Usually, the independent demand inventories are inventories for Finished goods which are prepared for final consumers. These types of inventories are usually based on the historical data from sales, seasonality and sales forecasts techniques. (“MSG Management Study Guide.”)

Dependent Demand Inventories.

Mentzer and Moon (2006) claim that dependent demand is demand for the components that are the parts of the final product. According (“MSG Management Study Guide.”) the dependent demand inventory is an inventory, which consists of the items that are dependent on another item or comes as a part of the final product. For instance, raw materials inventory and semi-finished parts inventory are the examples of the dependent demand inventories. For a better explanation of this sample, the instance of the bicycle could be viewed. Final consumer – the buyer of the bicycle defines the actual demand for the product. Hence, the bicycle is an independent demand item, and the enterprise, which produces these bicycles is holding independent demand inventory. However, bicycle consist of different components, if the final producer could not manufacture some of them, they are purchased from the outer suppliers. According to it, these components are dependent demand items, and their manufacturers hold dependent demand inventories. These kinds of inventories are managed through the MRP – Material requirements planning or ERP – Enterprise resource planning systems. Techniques that are needed for calculation of MRP or ERP data in its turn are based on different operational models such as Kanban, JIT (Just in time) or classical Inventory models such as EOQ (Economical order quantity). Nevertheless, the initial planning also relies on

such factors as sales forecast released for finished goods and other relevant historical data techniques. (“MSG Management Study Guide.”)

Independent vs Dependent Demand Inventories.

The differentiation of Independent and Dependent demand inventories is crucial since it requires implementation of different inventory control systems and operational models. According to (“MSG Management Study Guide.”) control of dependent demand, inventories is more sophisticated than control of independent demand inventories. Analysis of dependent demand system includes much more variables than analysis on independent demand systems. For instance: lead times, order quantities, delivery schedules, reorder points, coordinated capacities or schedules of logistics processes coupled with transit timelines of transportation and warehousing of dependent demand items before they are ready to be supplied to manufacturing facility are examples of open questions that has to be reviewed if an enterprise wishes to execute dependent demand inventory system (“MSG Management Study Guide”)

Nevertheless, the most fundamental and critical issues of dependent demand inventory systems are two questions or two variables that could also be considered as basic inventory decisions:

1. What is the optimal amount of order for each item?
2. At what particular time the order has to be placed?

The answers to these two questions are usually based on the analysis of two important variables: demand and lead time. (Ravinder; Misra 2014)

The researches aimed to reveal the instance of these questions are focused on work with different inventory models such as EOQ and inventory categorization techniques such as ABC analysis. In addition, the issue of the cost also plays a crucial role in the decision. (“MSG Management Study Guide”)

3.3 Inventory control

3.3.1 Inventory policies

Inventory policy is a standard operating principle, which defines the manner of inventory control and choice of appropriate inventory model. Due to it, a model of inventory is selected based on chosen inventory policy. In general terms, inventory policy is much wider definition than inventory model since it describes a framework of fundamental procedures that defines the core of operating principle, while inventory model stands for the particular technique or algorithm, which often assumes usage of certain formula and describes specific mechanism on the basis of which inventory control is implemented. (Vrat 2014, 29-33) Usually, inventory policy is represented in the form of a graph as a function of time. This graphical method provides better visualization and understanding of change in inventory status over a certain period and the point when new purchasing order is released. Vrat (2014, 29-33) defines three basic inventory policies:

1. Continuous Review Policy. Implementation of this inventory policy assumes that inventory levels are continuously monitored, and each new order is placed when the inventory level reaches reorder point (ROP). After the point is reached, the number of items (Q) is ordered. Very often, this amount is determined with the help of EOQ model, but considering inventory policy it does not have to correspond to that always. Generally, in Continuous Review System, two unknown variables exist: reorder point

(ROP) and fixed order quantity (Q), which corresponds to the question when to buy and what to buy. According to it, this policy could also be called (Q, R) policy. (Vrat 2014, 29-33) Graphically, CRP could be represented as follows:

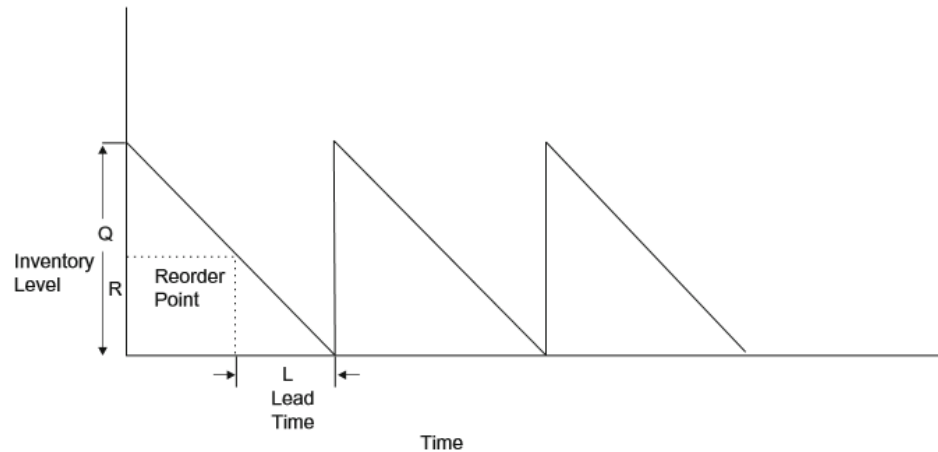


Figure 8 Continuous Review Policy vs Time. (Vrat 2014, 29-33)

2. Periodic Review Policy. The implementation of the system assumes that inventory is not monitored continuously; on the contrary, the review procedure happens after a fixed time interval (T). Each new order is placed whenever the review point is reached. The amount of order can be defined analytically according to the formula:

$$Q = \text{Order Quantity} = (S - X) \quad (9)$$

Where: S = order up to level (maximum stock level), X = is a stock on hand at the time of review. (Vrat 2014, 29-33)

In this case, we have two unknown variables to find S the order up to level and T a fixed time interval between two review points. According to it, the

PRP could also be called (S, T) policy. (Vrat 2014, 29-33) Figure 7 Graphically illustrates its principle:

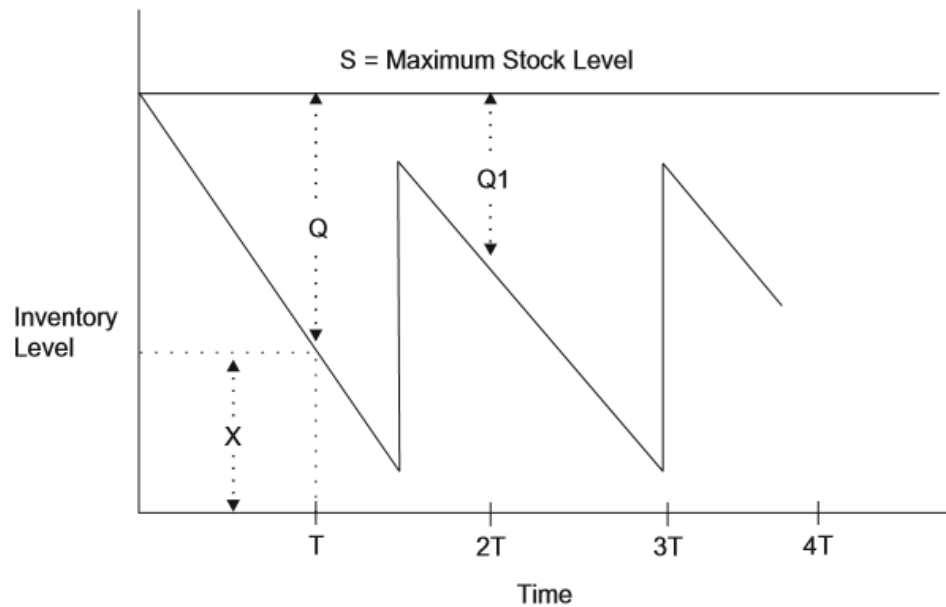


Figure 9 Periodic Review Policy vs Time (Vrat 2014, 29-33)

This replenishment policy is easy to implement, but it certainly has solid disadvantages. For example, when the review point is reached, but stock levels are still significantly high the new order has to be placed. This results in small order quantities and could lead to additional costs. (Vrat 2014, 29-33)

3. Optional Replenishment Policy. This type of inventory replenishment policy is aimed at optimization of order quantity based on periodical reviews. Two levels of inventory (X): minimum and maximum (s, S) respectively are established, and the stock levels (X) are monitored on a periodical basis with a fixed time interval (T). After the period (T) has ended, the stock levels (X) are checked and if they are still bigger than (s) no order is placed. In that case, replenishment is not initiated until next point of a review, as the inventory levels are sufficiently adequate to withstand another

replenishment cycle. However, if at the time of replenishment review the stock levels (X) are less than or equal to (s), a new replenishment order is placed. The quantity (Q) of replenishment order must correspond to the quantity that will increase stock levels (X) to the value of (S). (Vrat 2014, 29-33) Analytical representation of Optional Replenishment Policy is:

$$\begin{aligned} Q &= S - X \text{ if } X \leq s \\ &= 0 \text{ if } X > s \end{aligned} \quad (10)$$

Graphical representation of ORP:

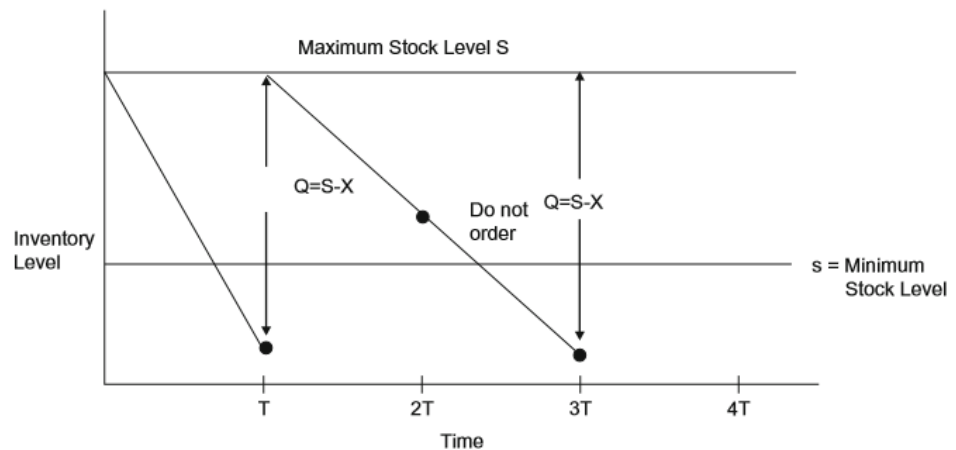


Figure 10 Optimal Review Policy (Vrat 2014, 29-33)

With this type of replenishment policy, the decision is based on the optimization of three variables s , S and T so this policy could also be called (s, S, T) replenishment policy. (Vrat 2014, 29-33)

Besides mentioned inventory policies there could be other options that differ from described ones. Nevertheless, these three models are the basic ones and most used in inventory management practices. Whether an organization wishes to implement certain inventory model to their operations, they first

have to decide which inventory policy would be sensible for their case. When the replenishment policy is employed, a decision about inventory model could be developed on its grounds. (Vrat 2014, 29-33)

3.3.2 ABC Analysis

ABC analysis is an item categorization technique, which is based on the 80/20 rule so called Pareto Principle that was developed by Italian Philosopher Vilfred Pareto and nowadays this method is widely used in inventory management. Items in inventory are divided into three classes A, B and C according to their usage value and cumulative percent share from stock. Items that refer to the 'A' category constitute the smallest number of items in inventory, but the highest usage value (80% of cumulative inventory). From a financial point of view, this category could be categorized as 'vital few.' Items in category 'B' constitute the medium number of items in inventory and refers to medium usage value (80-95% of cumulative inventory share) – from financial perspective defined as 'normal' items. Finally, category 'C' items obtain the highest number concerning inventory levels. However, it also refers to the category with the lowest value of usage (95-100% of cumulative inventory share) – 'the trivial many.' (Jessop & Morrison 1994, 156-159)

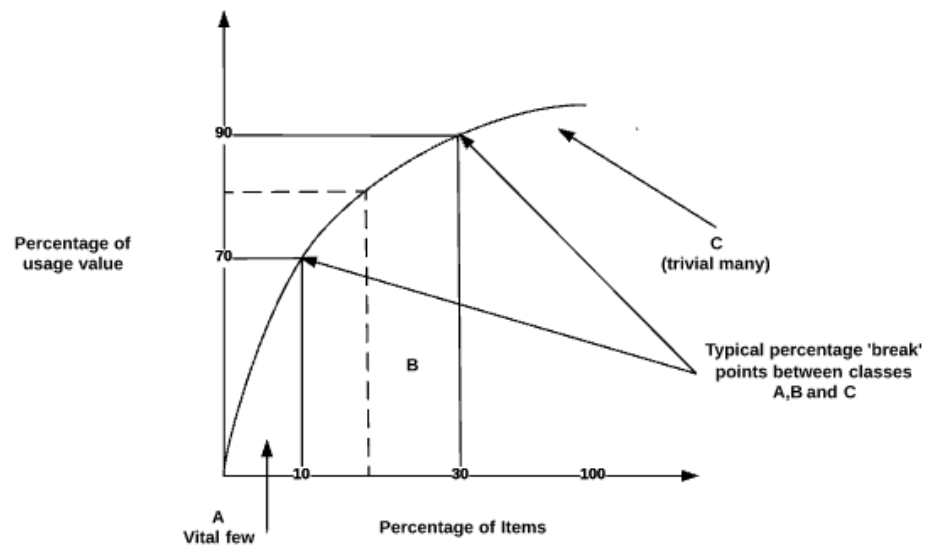


Figure 11 Typical ABC Curve showing the 80/20 relationship. (Jessop, Morrison 1994, 157)

According to Vollmann (1997, 722–725), one way to implement ABC Analysis is to establish criticality factor for the product value. Items with high critical value get number 1, items with no criticality receive number 3 and the rest of the items are marked with number 2. Consequently, nine new categories are established: A1, A2, A3, B1, B2, B3, C1, C2, C3. Afterwards, these categories are redefined to the three new groups: AA, BB, CC. Furthermore, according to the redefinition A1, A2 and B1 are handled through the category AA; A3, B2, C1 are handled through the category BB; and the rest B3, C2, C3 respectively handled through the category CC. Moreover, items that refer to category AA are considered as the critically important items, and they are replenished in small batches many times per year (usually 10-12), their safety stock is relatively big. Items of category BB are replenished according to a value that is received from EOQ calculation, the amount of replenishment times is around 6/year and the safety stock is considered to be relatively big. Finally, CC category items are handled as items with the smallest strategical importance; the replenishment is initiated a couple of times per year (usually 1-3) in huge batches, the safety stock is very small or not hold at all. (Vollmann 1997, 722–725)

Table 4 The ABC Analysis nine categories. (Adapted from Vollmann 1997, 722-725)

Category parameters			
	AA	BB	CC
Replenishment frequency	<i>Once a month</i>	<i>Half a year</i>	<i>Once a year</i>
Order Quantity	<i>Small</i>	<i>According to EOQ</i>	<i>Big</i>
Safety Stock	<i>Critically big</i>	<i>Critically big</i>	<i>Small or not hold at all</i>

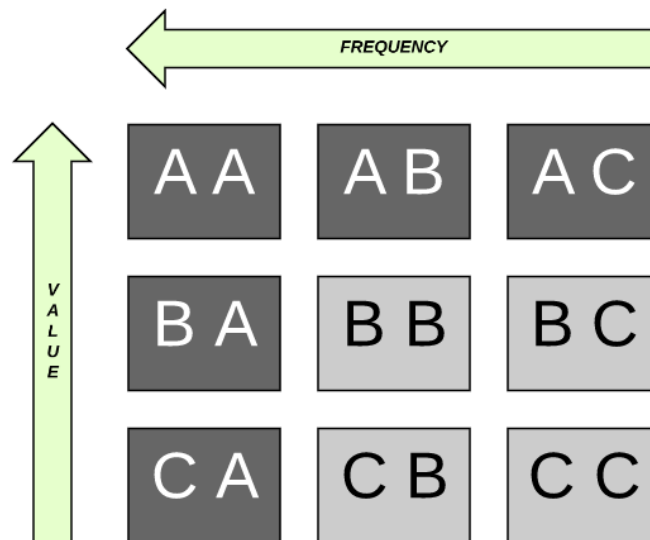


Figure 12 ABC Analysis classification according to Value and Frequency according to Gwynne (2011, 62-64).

Key insight into ABC Analysis.

The first application of ABC analysis categorization technique was created at 1950's. From that day on, ABC analysis has been considered as the only one "top-notch" categorization method in inventory management. However, industrial world is evolving in exponential manner: what was innovative and relevant 70 years ago became inessentially obsolete nowadays. One of the

main reasons to use ABC analysis was the lack of Information technologies and the limited capacity in data processing and handling. Nowadays, the development of IT (Information technology) has a crucial influence on the technology-based business, and the availability of computers eliminates the need to use ABC analysis. (Spearman 2015)

As it appears, this approach is relatively limited and cannot be applied to many specific item categories. For example, its irrelevance could be best demonstrated in make-to-order assembly system that assumes handling of components with long lead-time value. In this case, it is sensible to define factors that influence the amount of inventory: demand and lead-time. According to the pattern of demand and type of item, inventory could be divided in two ways: 1) high and low demand inventories 2) standard and custom items inventories. Standard items are classified as items with sufficiently high demand and shorter lead times. These factors are significant enough to provide relatively high stock levels of this category. On the contrary, custom items are those, which have low demand and high value. Empirically, it was seen that customer does not expect a supplier to have this category in inventory. Consequently, taking into consideration all the attributes new four categories could be defined: 1) High demand, Long lead time 2) High demand, Short lead time 3) Low demand, Long lead time 4) Low demand, Low lead time.

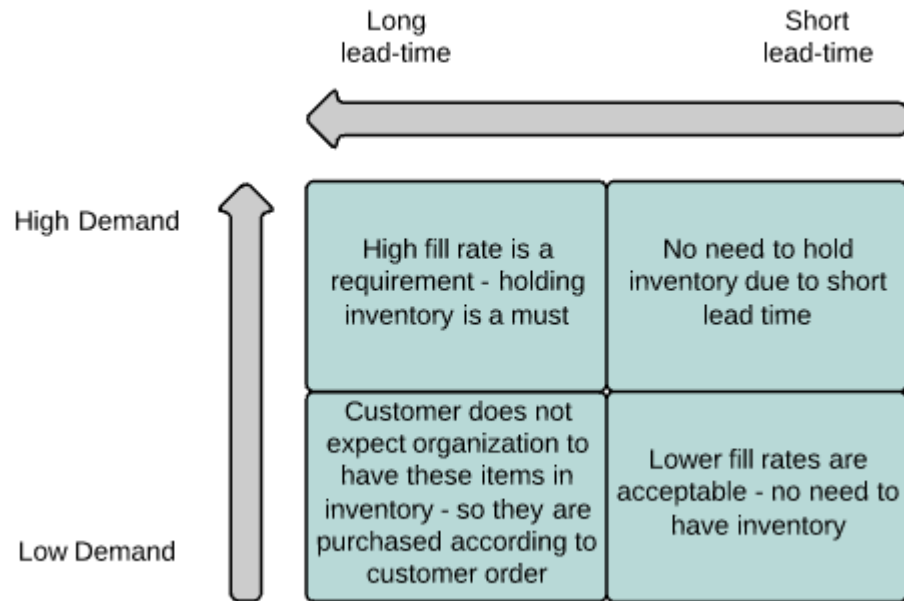


Figure 13. Item categories in make-to-order assembly system according to Spearman (2015)

If the product lead time is short and most probably shorter than lead time agreed with the customer, there is no need to hold inventory. Items with low demand and low lead time are allowed to have lower fill rates to some extent if the backorder time is manageable. Thus, the only category that needs high fill rate is High demand items with long lead time. This method provides new knowledge and approach on inventory categorization and can significantly reduce stock levels. Moreover, the defined categories are very different from standard ones A, B, C and could not be managed through the application of the basic method. Nowadays, up-to-date software and modern computers could perform optimization of multiple variables such as demand and lead time in conjunction with each individual component even if the number of components is high. At this point, ABC classification is considered as an impractical approach, since it implies inventory division into three standardized degrees rather than seeks for substantial, complicated diversity among these categories. The reason, why ABC analysis is still highly popular, is probably the fact that software suppliers tend to include it in their

products as the easiest and familiar method. Nevertheless, familiarity and simplicity are not manners to solve complicated problems. (Spearman 2015)

3.3.3 Economical Order Quantity model

Economical order quantity model is recognized as one of the first mathematical applications to operations management. The EOQ model and its formula were developed by Ford Whitman Harris in 1913. Initially, Harris was trying to find a specific algorithm that will define an optimal lot size in purchasing or production order and also determine the factors which would influence the optimal lot size. The general perplexity that guided Harris through his research was the question “How many parts to make/purchase at once to perform cost effective operations?” (Hopp & Spearman 2008, 50-51)

According to Harris (1913, 135-136), the maximum quantity of production/order to be optimally utilized at a time is determined by interest on capital tied up in salaries, material and overhead. The minimum is determined by set up cost, which includes: labor and material cost to prepare a workshop to manufacture the product. Similarly, if the question is purchased lot size, set up cost is associated with the sum of the cost of placing a purchase order. (Hopp & Spearman 2008, 50-51)

As the current research implies the development of Inventory model, the EOQ definition is slightly different from the original one, since original purpose was to find the optimal production lot. According to it, the EOQ is represented as the algorithm for finding the optimal purchase lot that goes to inventory.

To define the Economical Order Quantity model, several assumptions have to be made:

- 1) Demand is known, there is no uncertainty about it.
- 2) Lead time of delivery is zero.

- 3) Stock utilization is instantaneous.
- 4) Demand is continuous over time horizon.
- 5) Every time an order is placed, fixed cost is incurred. Fixed cost is always equal neglecting the size of order.
- 6) It is assumed that there are no interactions between product and they could be analyzed individually.

Adapted from (Hopp & Spearman 2008, 50-51) and (Muckstadt & Sapro 2010, 17-42)

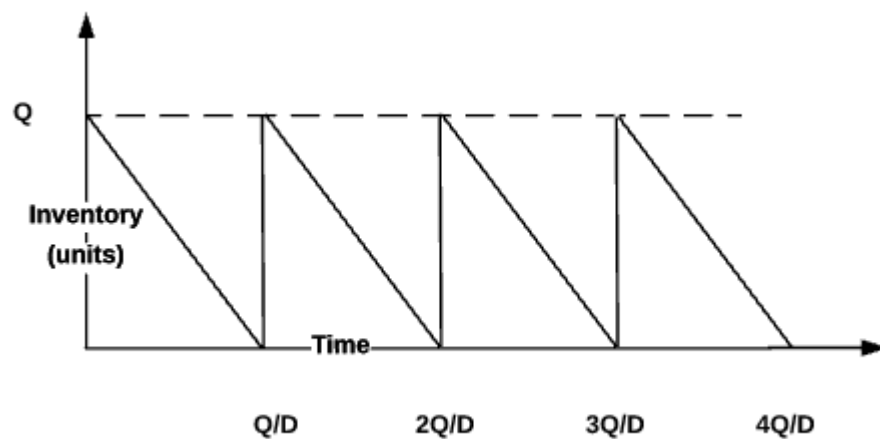


Figure 14 Inventory versus time in EOQ model. (Hopp & Spearman 2008, 51)

As Harris (1913) assumed that demand is defined as continuous and known, according to Figure 14 the average inventory level is calculated as $Q / 2$ since Q number of units is ordered when the inventory level reaches zero value. Let the holding cost be calculated as $i * c = h$, then holding costs of inventory results in $h * Q / 2$ annually. The amount of orders to satisfy demand per year is computed as D / Q , according to it, the annual fixed ordering cost is $A * D / Q$. Finally, summing up the derived equations, the total cost, which constitutes the annual fixed order cost and inventory holding cost per year is $Y(Q)$ is represented by equation (11). (Hopp & Spearman 2008, 52)

$$Y(Q) = \frac{h \times Q}{2} + \frac{A \times D}{Q} \quad (11)$$

Graphical representation of the total costs function:

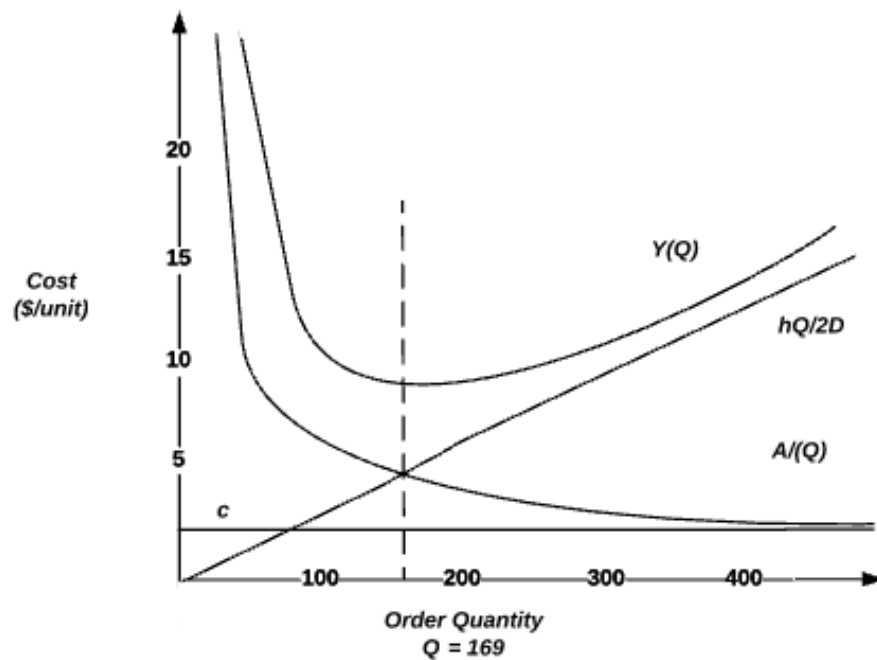


Figure 15 Costs' functions in EOQ model. (Hopp & Spearman 2008, 52)

To get a better understanding of the model, it is useful to explore it graphically. According to Figure 15, the annual cost associated with holding inventory ($h \times Q / 2$) increases linearly as the order quantity (Q) increases. On the other hand, the annual fixed ordering cost ($A \times D / Q$) decreases, indicating that placing a larger amount of order (Q) results in significantly

reduced annual ordering costs. Furthermore, certain optimal order quantity Q reduces the total cost $Y(Q)$ to the optimized minimum value exactly at the value of Q , where two cost functions $(h * Q / 2)$ and $(A * D / Q)$ are intersecting each other, indicating the smallest, optimal total cost in accordance with EOQ. Simplistically, the EOQ defines the minimized function of total cost with (Q) value set at the balance point between the cost of holding inventory and annual fixed order cost. (Hopp & Spearman 2008, 52)

Finally, to find the minimum value of total cost function $Y(Q)$, it is necessary to take the derivative of equation (11) with respect to Q and establish the result equal to zero. (See Hopp & Spearman (2008, 53) for derivation)

Accordingly, the optimal order quantity Q that minimizes the values of equation (11) is computed according to the formula:

$$EOQ = Q = \sqrt{\frac{2 \times A \times D}{i \times c}} \quad (12)$$

Where: D = demand (units/year); c = unit cost; A = fixed order cost to make a purchase order; i = annual interest rate; Q = the decision variable, which represent lot size in units.

Analyzing the EOQ model.

The consequence derived from the EOQ model is the notion that the amount of optimal order quantity increases with a square root of fixed ordering cost or rate of demand and decreases with a square root costs associated with inventory. In addition, another significant consequence is namely the fact that increase in the lot size also magnifies the amount of inventory on hand, while simultaneously decreases ordering frequency (Hopp, Spearman 2008,

52). Nevertheless, the essential questions that arise from the implementation of EOQ square root formula are 1) “How reliable is the result obtained from EOQ calculation?” 2) “Does the formula considers all relevant factors, affecting the order quantity?”

According to (Jonsson 2008, 433) Mathematical computation of EOQ formula indeed produces the optimal result, as it derives sum of holding cost and fixed ordering costs closer to zero. However, the mathematical result from EOQ formula is totally dependent on the modeling assumptions, some of which require critical analysis. (Hopp & Spearman 2008, 53-55) For example, the model assumes that demand pattern is continuous through the year and utilization of stock is instantaneous: does this statement realistic enough to be accounted as a basic truth? What will happen if the demand will fluctuate during the year? In that case, it is wise to adjust the order quantities according to demand curve; otherwise, there is a high probability of the stock out. Another assumption of EOQ is that lead-time in equation (12) is not taken into account, therefore it is either considered to be zero or equal for every order placed. In reality, this statement is very questionable; since in practice lead-time is almost always accounts for its variability and actually, suppliers are experiencing delays quite often. In this situation, stock out situation is possible since the amount of inventory-on-hand is minimum and reorder points, which take into account lead-time variability, are not established. Besides, the application of EOQ principles implies the fact that holding cost for each inventory unit is predicted with calculation according to the steady consumption until they units will be ordered again. What will happen, if all units would be utilized before predicted end of replenishment cycle or vice a versa, they will not be consumed until the reorder point? The projected accounting records in the first case will show less profit for the organization, in the second case in will result at over stock and extra expenses related to it. (“The Advantages & Disadvantages of Economical Order Quantity.”) Another issue about EOQ model is discussed by Ford W.

Harris. (1913, 135-136) He claims that one the most sensitive parts is the computation of the setup cost or ordering cost. In reality, estimation of fixed ordering costs is usually a difficult task, and if the prediction is wrong, the results obtained from EOQ calculation will not correspond to the actual values. (Hopp & Spearman 2008, 52) Nevertheless, in spite of its flaws, EOQ has proven to be robust and reliable. Empirically, it has been seen that model could produce nearly optimal results in the simple cases when assumptions about the model are not violated and more or less correspond to the truth. However, the reality is not associated with simplicity, and consequently, there is a requirement for more sophisticated, profound solutions. (Hillier & Lieberman 2010, 942-955)

3.3.4 Statistical Inventory models

The approaches discussed in the previous section consider simple cases when the approximations are clearly defined, and demand is known beforehand. Statistical inventory models are a manner to solve more complicated problems when the demand is uncertain and computational algorithm assumes estimation of two decision variables: statistical reorder point, statistical order quantity. (Hopp & Spearman 2008, 66-67)

3.3.5 Base stock level model

The base stock level model is statistical inventory model that based on the continuous review framework. The method initiates replenishment with one unit at a time, when inventory position reaches certain point and demand occurs randomly. The model primarily answers the key question “if the storing space is limited, costs have to be minimized and lead time of supply is long enough to eliminate the need of having inventory, how much stock to carry to avoid stock out situations?” (Hopp & Spearman 2008, 71-72) To create the model, we have to set following assumptions about its parameters:

- 1) There are no interactions between products, and they could be analyzed independently.
- 2) Batch ordering is not possible, and demand comes one at a time.
- 3) Sales are not lost, but backordered.
- 4) There is no uncertainty concerning lead-time variability. Lead time value is known.
- 5) There is no fixed ordering cost or constraint on the number of replenishments.
- 6) Demand is predicted through continuous distribution.

(Hopp & Spearman 2008; 71-72)

Consequently, the model's parameters:

L = replenishment lead time (days)

X = random variable, representing the demand during lead time

$g(x)$ = probability density function of demand during lead time

$G(x) = P(X \leq x)$ = cumulative distribution function of demand during lead time

$\theta = E(X)$ = mean demand (units) during lead time

σ = standard deviation of demand during lead time

h = cost to carry one unit of inventory during 1 year (€/unit)

b = cost to carry one backorder unit during 1 year (€/unit)

r = reorder point when the replenishment procedure has to be initiated (units)

$ss = r - \theta$, safety stock (units)

$S(r)$ = service level or fill rate, representing the percentage of orders that completed from stock as a function of r

$B(r)$ = average amount of backorders as a function of r

$I(r)$ = average level of inventory on hand (cycle stock) as a function of r (units)

(Hopp & Spearman 2008, 71-72)

The main idea of the base stock level model is to find reorder point r , which determines the inventory level when the replenishment order has to be initiated. Inventory on hand refers to the amount of stock; backorders refer the amount of unfilled demand that will be filled from coming orders; orders refer to the upcoming supply from vendors, and the inventory position is established as:

$$\begin{aligned} \text{Inventory position} \\ = \text{on - hand inventory} - \text{backorders} + \text{orders} \end{aligned} \quad (13)$$

(Hopp & Spearman 2008, 71-72)

Graphical representation of inventory position versus time:

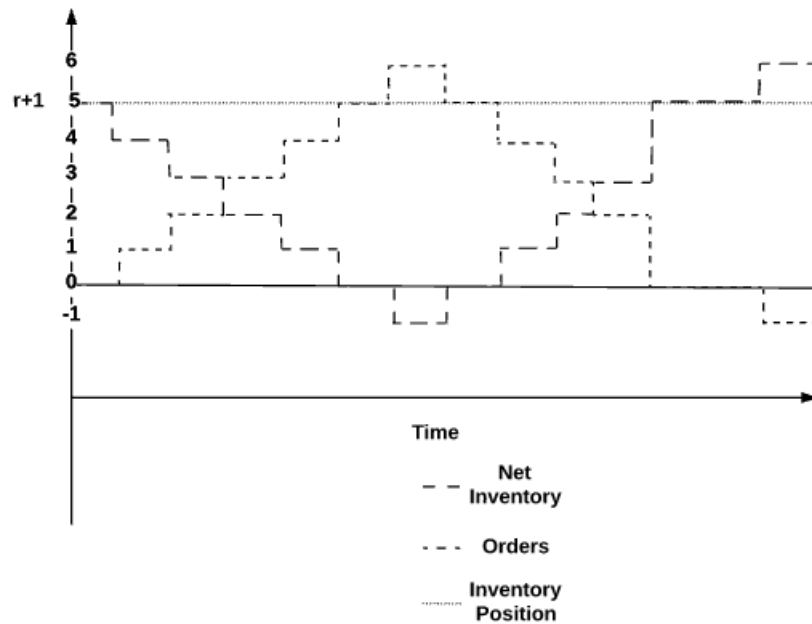


Figure 16 Inventory position in Base Stock level model. (Hopp & Spearman 2008, 73)

The general rule of base stock system is to maintain base stock level $r + 1$ and initiate replenishment always when inventory reaches r point.

Therefore, when inventory level reaches r , the replenishment order is placed, and it takes L lead-time to arrive. The mean demand L is supposed to be θ , accordingly, the inventory level when an order arrives is expected to be $r - \theta$. Thus, if the $r - \theta > 0$ it is defined as safety stock. Hence, the main decision variable is r , but generally, two more values have to be defined safety stock = $r - \theta > 0$ and base stock level = $r + 1$. (Hopp & Spearman 2008, 73)

The model development is possible to initiate in two ways: either cost function that minimizes reorder point could be built, or specific fill rate could be established, and reorder point according to service level is computed. It is useful to use both of these methods and check how they correlate and validate each other. However, firstly the performance measures $S(r)$, $B(r)$, $I(r)$ have to be derived. (Hopp & Spearman 2008, 73)

The derivation could be computed from the following equation:

$$\text{Inventory position} = r + 1 \quad (14)$$

(Hopp & Spearman 2008, 73)

Service level $S(r)$.

According to Hopp & Spearman (2008, 73), service level is a specific measure of inventory control, which represents the percentage of customer demand filled from stock, also expressed as customer service level. In base stock level model, the service level is derived as:

$$S(r) = G(r + 1) \quad (15)$$

(Hopp & Spearman 2008, 74)

It is assumed that all orders are the same and purchased with one unit at a time. Consequently, when we place replenishment order, our inventory position is equal $r + 1$, including inventory on hand + inventory coming from order. As lead time is considered to be continuous, it is known that all the other r items on-hand or on order could be used to fill new demands before an order under consideration arrives. (Hopp & Spearman 2008, 74)

Accordingly, the probability that order will arrive after its' demand happened only if the demand during lead time θ is higher than $r + 1$ or equal to it: $P(X \geq r + 1)$. (Hopp & Spearman 2008, 74) Thus, the probability that order comes before it's demand is opposite:

$$1 - P(X > r + 1) = P(X < r + 1) = G(r + 1) = S(r) \quad (16)$$

(Hopp & Spearman 2008, 74)

If demand is normally distributed, then the fill rate could be expressed as:

$$S(r) = G(r + 1) = \Phi \left(\frac{r + 1 - \theta}{\sigma} \right) \quad (17)$$

(Hopp & Spearman 2008, 74)

where Φ is cumulative probability distribution function. (Hopp & Spearman 2008, 74)

Backorder level $B(r)$.

Hopp & Spearman (2008, 74) claims that term backorder is a term used in inventory control that refers to the customer order, which has not been filled from stock and would be filled when new replenishment order will arrive. In the base stock level model, the number of orders (X) is precisely equal to the number of demands during lead time L .

From the Inventory position equation (13) and (14) the following is derived:

$$\text{On hand inventory} - \text{backorders} = r + 1 - X \quad (18)$$

(Hopp & Spearman 2008, 74)

According to it, inventory on hand and backorders could not be positive at the same time, since either the inventory on hand is consumed or the backorders are filled. Thus, when the number of outstanding orders is $X = x$, backorder level is defined:

$$\text{Backorders} = \begin{cases} 0, & \text{if } x < r + 1 \\ x - r - 1, & \text{if } x \geq r + 1 \end{cases} \quad (19)$$

(Hopp & Spearman 2008, 74)

Therefore expected backorder level could be calculated through the integration of function $(x - r - 1)$ by finding an average of all possible values that x can take:

$$B(r) = \int_{r+1}^{\infty} (x - r - 1)g(x)dx = \int_r^{\infty} (x - r)g(x)dx \quad (20)$$

The formula is obtained from (Hopp & Spearman 2008, 74) but for the complete derivation see (Zipkin 2000).

If the demand is normally distributed, then it could be derived as follows:

$$B(r) = (\theta - r) \times [1 - \Phi(z)] + \sigma \times \phi(z) \quad (21)$$

where: $z = (r - \theta) / \sigma$, Φ = cumulative probability distribution function, ϕ = probability density function. (Hopp & Spearman 2008, 75)

Inventory level I(r).

Inventory on hand is represented as $I(r)$, backorder level is $B(r)$ and $\theta = E(X)$ demand during lead time. (Hopp & Spearman 2008, 75) Taking into account the equation:

$$\text{On hand inventory} - \text{backorders} = r + 1 - X \quad (22)$$

Inventory position is derived:

$$I(r) = r + 1 - \theta + B(r) \quad (23)$$

(Hopp & Spearman 2008, 75)

The approach of cost function and base stock level optimization.

The other way to compute base stock level is to derive the cost function, which defines reorder point that minimizes total cost. (Hopp & Spearman 2008, 76) The total cost function is considered to be the sum of inventory holding cost and backorder cost:

$$\begin{aligned} Y(r) &= \text{holding cost} + \text{backorder cost} \\ &= h \times I(r) + b \times B(r) \\ &= h(r + 1 - \theta + B(r)) + b \times B(r) \\ &= h \times (r + 1 - \theta) + (b + h) \times B(r) \end{aligned} \quad (24)$$

(Hopp & Spearman 2008, 76)

Further, the reorder point that decreases total cost function to the minimum is computed as derivative from $Y(r)$ total cost function:

$$\frac{dY(r)}{dr} = h + (b + h) \times \frac{dB(r)}{dr} \quad (25)$$

Therefore, the equation $B(r) = \int_{r+1}^{\infty} (x - r - 1)g(x)dx$ has to be differentiated and from that $\frac{dB(r)}{dr}$ is computed:

$$\begin{aligned}\frac{dB(r)}{dr} &= \frac{d}{dr} \int_{r+1}^{\infty} (x - r - 1)g(x)dx \\ &= - \int_{r+1}^{\infty} g(x)dx \\ &= - [1 - G(r + 1)]\end{aligned}\tag{26}$$

Afterwards, the total cost function $Y(r)$ is set to be zero, accordingly:

$$\frac{dY(r)}{d} = h - (b + h) \times [1 - G(r + 1)] = 0\tag{27}$$

(Hopp & Spearman 2008, 76)

Solution of the previous expression will formulate new equation, which computes the reorder point r that minimized total cost function $Y(r)$:

$$G(r^* + 1) = \frac{b}{b + h}\tag{28}$$

The formula is obtained from (Hopp & Spearman 2008, 77) but for the complete derivation see (Zipkin 2000).

Summing up base stock level model.

The general features obtained from analysis of base stock level model are:

- 1) Reorder point establish base stock level and safety stock that buffers against stock out situations.

- 2) Base stock level is an increasing function of the mean demand and standard deviation of the demand during lead time.
- 3) The base stock level is possible to define through the constraint of service level or backorder cost since the optimal fill rate is an increasing function of backorder cost and a decreasing function of holding cost.

(Hopp & Spearman 2008, 78)

Finally, the base stock level model is a useful solution when the quantity of the order is not the key question, and main decision variable is reorder point r or base stock level to keep (Hopp & Spearman 2008, 78). However, what solution would be advisable to use, if the question is two decision variables: quantity of order and reorder point?

3.4 The (Q, r) Model

The (Q, r) model is a sophisticated statistical inventory model, based on continuous review system. However, apart from base stock level model and EOQ model, the (Q, r) model conjoins two decision variables that were separated in the two previously described models: r = reorder point, which represents the time when replenishment order has to be placed and base stock level (how much stock to keep); Q = quantity of material or component that has to be ordered at a time. The essence of (Q, r) model is to find the decision values. (Hopp & Spearman 2008, 78-79)

The assumptions concerning model's parameters are similar to the ones that have been made about the Base Stock Level Model, although two more assumption has to be considered:

- 1) There is a fixed ordering cost that occurs every time the order is placed.
- 2) The number of possible orders per year is restricted.

Accordingly, the replenishment which is one unit at a time is not possible to initiate, as it has been in the Base Stock Level model. (Hopp & Spearman 2008, 78-79)

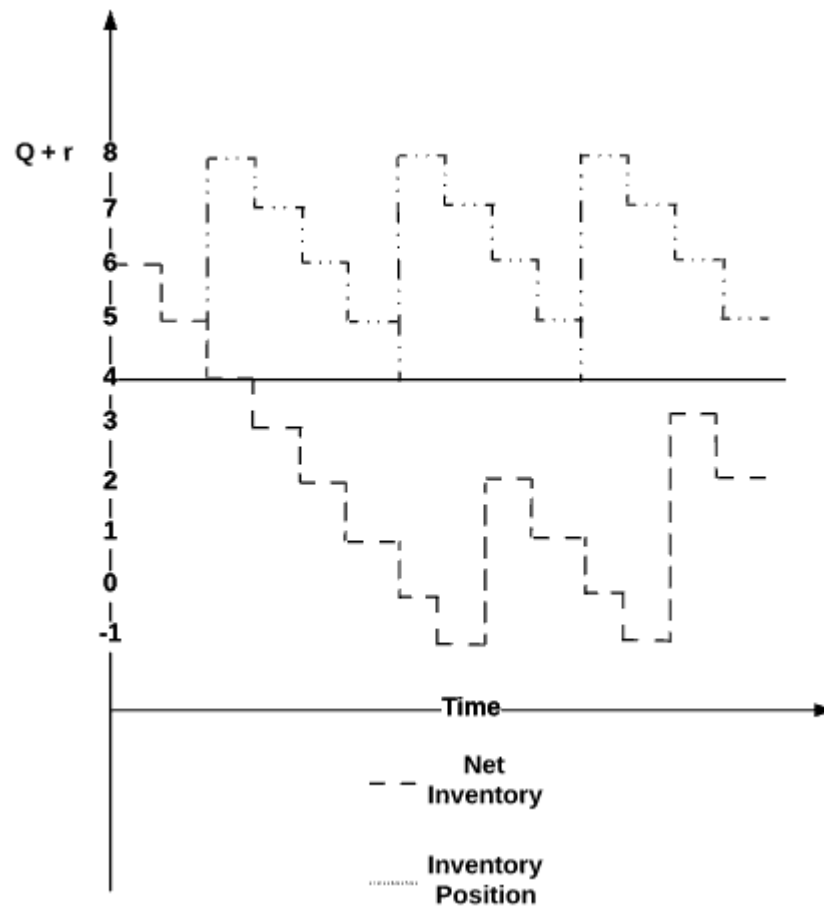


Figure 17 Inventory position in the (Q, r) model. (Hopp & Spearman 2008, 79)

The graphical representation of the continuously monitored unit in the (Q, r) model could be seen in Figure 17. Demand for the product is randomly distributed, and it is assumed that it happens one at a time, each time when the product is ordered inventory position drops one level down. When the stock level reaches the reorder point r , the new replenishment order Q is placed, and after fixed lead-time L it arrives in inventory. It is important to notice, that lead time pattern is assumed to be fixed and known, identically with Base Stock level model, however, the (Q, r) is possible to extend to more complex assumptions and implement concept of lead-time variability. All in all, the main focus of the (Q, r) model is to determine the optimal order

quantity and the amount of inventory to carry. (Hopp & Spearman 2008, 78-79)

One of the key insides of (Q, r) model is the correlation of Q and r control values. In previous examples of EOQ and Base stock level model, Q and r were completely separated from each other and represented different objectives. For example, in the EOQ order quantity Q represented the balance between the ordering frequency and the amount of inventory on-hand. The larger is the value of Q ; the smaller is the replenishment frequency and Higher average stock levels. Vice a Versa, if the average inventory level is small, then Q obtains smaller value but results in higher replenishment frequency. On the contrary, the r value in the base stock level model represents the tradeoff between the probability of stock out and base stock level. For instance, the increasing value of r is decreasing the probability of a stock out, but increases the base stock level and hence inventory costs. Otherwise, the lower reorder point computes lower base inventory levels and costs, but the stock out expectancy is higher. In the (Q, r) these tradeoffs will correlate to each other in terms of inventory effects, ordering frequency, and service level, depending on how cost functions and service level are set. In addition, the relevant distinction of the core between Q and r is the fact that they represent completely different stock components. In the EOQ the Q refers to the cycle stock and r in the Base Stock Level model refers to the safety stock, while the (Q, r) model defines the conjunction of these two models. (Hopp & Spearman 2008, 78-79)

The (Q, r) model is possible to develop in two ways: Backorder cost approach and Stock out cost approach. The difference between them is the representation of customer service level. Backorder cost approach assumes that if the customer demand is unfilled, the penalty per unit time is established. Stock out cost approach is initiated in the way that fixed penalty is given to each demand that was not filled. (Hopp & Spearman 2008, 78-79)

To develop the both approaches, firstly two cost functions from EOQ and Base stock level models have to be established:

$$\text{Backorder cost: } \min(Q, r) \{ \text{fixed setup cost} + \text{backorder cost} + \text{holding cost} \} \quad (29)$$

$$\text{Stock out cost: } \min(Q, r) \{ \text{fixed setup cost} + \text{stockout cost} + \text{holding cost} \} \quad (30)$$

(Hopp & Spearman 2008, 80)

Further, the model's parameters are defined as:

D = expected demand (units/year)

L = replenishment lead time (days)

X = random variable, representing the demand during lead time

$g(x)$ = probability density function of demand during lead time

$G(x) = P(X \leq x)$ = cumulative distribution function of demand during lead time

$\theta = E(X) = D \cdot L / 365$ = mean demand (units) during lead time

σ = standard deviation of demand during lead time

h = cost to carry one unit of inventory during 1 year (€/unit)

b = cost to carry one backorder unit during 1 year (€/unit)

A = purchase order cost per one replenishment (euros)

c = unit production cost (euros)

k = stock out cost (euros)

Q = optimal order quantity (units)

r = reorder point when the replenishment procedure has to be initiated (units)

$ss = r - \theta$, safety stock (units)

$F(Q, r)$ = ordering frequency as function of Q and r

$S(Q, r)$ = service level or fill rate, representing the percentage of orders that completed from stock as a function of Q and r ($S(1, r) = S(r)$ = Base stock level fill rate)

$B(Q, r)$ = average amount of backorders as a function of Q and r ($B(1, r) = B(r)$ = Base stock backorder level)

$I(Q, r)$ = average level of inventory on hand (cycle stock) as a function of Q and r (units) ($I(1, r) = I(r)$ = base stock inventory level)

(Hopp & Spearman 2008, 80)

Costs.

Fixed ordering cost.

Generally, in the (Q, r) model it is essential to have order quantity more than 1 unit at a time. Otherwise, the essence of (Q, r) model will be similar to the base stock level model and the need for calculation of second decision variable Q will be eliminated. (Hopp & Spearman 2008, 80) Replenishment frequency could be restricted with a number of orders per year and computed according to this formula:

$$F(Q, r) = \frac{D}{Q} \quad (31)$$

Accordingly, $Q = \frac{D}{F(Q, r)}$

another option is to charge fixed cost every time, a replenishment order is placed, then the mathematical computation is:

$$F(Q, r) \times A = \frac{D}{Q} \times A \quad (32)$$

(Hopp & Spearman 2008, 80)

Cost of stock out.

Stock out cost approach is one of the possible solutions to charge poor customer service. According to Hopp & Spearman (2008, 80), the annual cost of stock out is directly dependent on the average amount of stock outs in a year: $D[1-S(Q,r)]$. Further, looking at the graph of (Q, r) (Figure 17) model over the time range it could be seen that inventory position is a plot between values of r and $r+Q$. Actually, in long term perspective, it will be seen that inventory position could take any value within the limit of r and $r+Q$ since it is distributed uniformly. (Hopp & Spearman 2008, 80)

If it is assumed that when one looks at the system after a certain operational period and the inventory position is equal to x , it means that inventory contains enough items to be able to fill next x units of demand. In this situation, the one might ask, what is the probability that demand equal to $(x+1)$ units could be filled from inventory? Analogically with BSLM, as all outstanding orders that will arrive within the range of replenishment lead-time L , the $(x+1)$ demand could be filled from inventory subject to mean demand θ is less or equal to zero. (Hopp & Spearman 2008, 81) (For analogy with BSLM see page 78, equation (16).)

In appliance with the BSLM the resulting probability formula:

$$P\{X \leq x\} = G(x) \quad (33)$$

Where: x is inventory position in a certain period, X is the mean of lead time that could be filled from stock and not resulted in stock out. $G(x)$ is cumulative distribution function of demand during lead time, which represents the likelihood that the demand during lead time (X) is less or equal to inventory position (x) and would be filled out from stock. (Hopp & Spearman 2008, 81) Subsequently, the probability that stock position can take any value within the limit r and $r + Q$ is equal for all options and service

level formula is formulated through the integration of all values between r and $r + Q$:

$$S(Q, r) = \frac{1}{Q} \times \int_r^{r+Q} G(x) dx = 1 - \frac{1}{Q} [B(r) - B(Q + r)] \quad (34)$$

The formula is obtained from (Hopp & Spearman 2008, 81) but for the complete derivation see (Zipkin 2000).

Important note is that function of $B(r)$ is similar to the one that was derived in the base stock level model. (Hopp & Spearman 2008, 81)

Nevertheless, Hopp & Spearman (2008, 82) claim that implementation of service level $S(Q, r)$ formula could be complicated to apply to optimization models. However, two types of approximations: type 1 and type 2 are suggested for the simplification.

Type 1 service approximation is formula for service level obtained from the base stock level model: $S(Q, r) \approx G(r)$ is tending to underestimate the actual service level due to the fact that this is cumulative probability function of demand (X) during lead time and consequently the increasing function of inventory position x . The advantage of this approximation is that it provides a relatively simple way of fill rate computation since it deals only with the value of r and excludes the value of Q . (Hopp & Spearman 2008, 82)
However, Hopp & Spearman (2008, 82) claim that it could significantly underrate the actual fill rate.

Type 2 service approximation is computed by disregarding the second variable in the derived service level expression for (Q, r) model (34):

$$S(Q, r) \approx 1 - \frac{B(r)}{Q} \quad (35)$$

This approximation concerning the service level is analogically underrate the actual fill rate. However, it is much more complicated, since it involves values

of both variables Q and r , as it was seen in the original formula (34). (Hopp & Spearman 2008, 82)

Cost of backorder.

Backorder cost in another approach to charge the poor customer service. According to Hopp & Spearman (2008, 82) the average level of backorders $B(Q, r)$ is proportional to the annual cost of backorder and it is equation could be formulated in the same way that equation for the service level: through the integration of all possible values that x could take within the limit r and $r + Q$.

$$B(Q, r) = \frac{1}{Q} \times \int_r^{r+Q} B(x + 1) dx \quad (36)$$

The formula is obtained from (Hopp & Spearman 2008, 82) but for the complete derivation see (Zipkin 2000).

The simplification of the formula (36) to the further function is taken from Hopp & Spearman (2008, 82) and could be used for normal distribution only:

$$\begin{aligned} \beta(x) &= \int_x^{\infty} B(y) dy \\ &= \frac{\sigma^2}{2} \times \{(z^2 + 1) \times [1 - \Phi(z)] - z \times \phi(z)\} \end{aligned} \quad (37)$$

where $z = (x - \theta)/\sigma$

(The formula is obtained from (Hopp & Spearman 2008, 82) but for the complete derivation see (Zipkin 2000).)

This allows to derive simpler equation of $B(Q, r)$:

$$B(Q, r) = \frac{1}{Q} \times [\beta(r) - \beta(r + Q)] \quad (38)$$

The formula is obtained from (Hopp & Spearman 2008, 82) but for the complete derivation see (Zipkin 2000).

Analogically, Hopp & Spearman (2008, 82) claim that it is useful to use an approximation of $B(Q, r)$ that does not include the value of Q . Similarly with type 1 service of $S(Q, r)$ it is possible to apply backorder formula from base stock level:

$$B(Q, r) \approx B(r) \quad (39)$$

Holding cost.

One more cost that has to define for the (Q, r) model is the inventory holding cost that could be computed as product of inventory holding cost per unit (h) and average inventory on hand $I(Q, r)$: $h \times I(Q, r)$. (Hopp & Spearman 2008, 83)

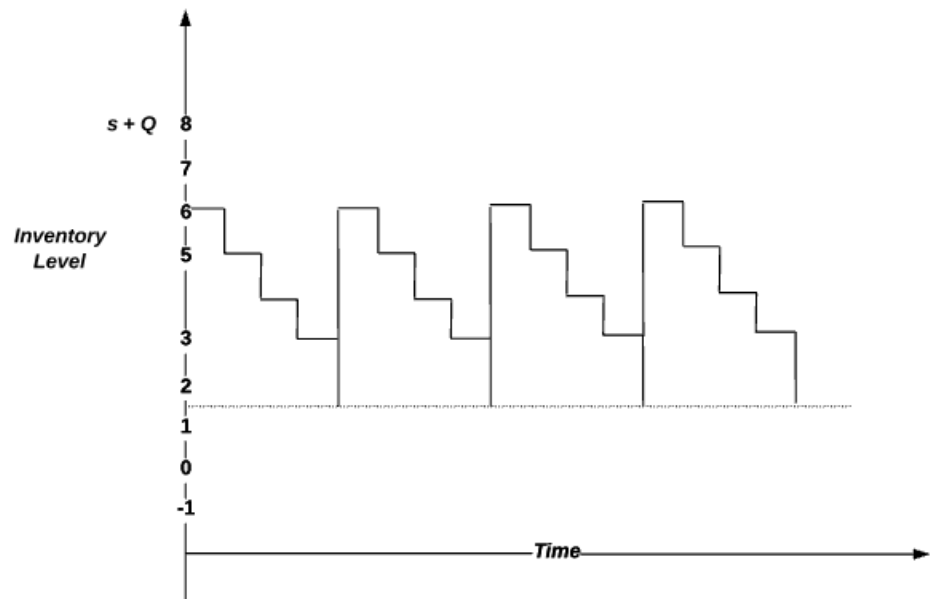


Figure 18 Expected Inventory position in the (Q, r) model. (Hopp & Spearman 2008, 83)

From the Figure 18 it can be seen that $Q = 4$, $r = 4$, $L = 2$ and $\theta = 4$. Demand is constant, definite and when the stock level reaches the reorder point $r = 4$, it takes fixed lead time $L = 2$ to arrive. Accordingly, the lowest inventory level is computed as $r - \theta + 1 = 4 - 4 + 1 = 1$. (Hopp & Spearman 2008, 83)

Subsequently, the inventory position is plot between the range of values $Q + ss$ and $ss + 1$. Average inventory level equation is formulated:

$$I(Q, r) \approx \frac{(Q + ss) + (ss + 1)}{2} = \frac{Q + 1}{2} + ss = \frac{Q + 1}{2} + r - \theta \quad (40)$$

(Hopp & Spearman 2008, 83)

Nevertheless, it is useful to remember that in reality demand's curve almost never takes the form as in Figure 18 and due to that fact, it may cause backorders. Consequently, Hopp & Spearman (2008, 83) claim that $I(Q, r) < 0$ is not possible and the equation above underrates the actual average inventory according to the backorder level and so the precise formulation is:

$$I(Q, r) = \frac{Q + 1}{2} + r - \theta + B(Q, r) \quad (41)$$

(Hopp & Spearman 2008, 83)

Backorder cost approach.

When the costs of initial equations (29) are defined, it is possible to derivate total cost function concerning two approaches. The cumulated equation of total cost function according to backorder cost approach is:

$$Y(Q, r) = \frac{D}{Q} \times A + b \times B(Q, r) + h \times I(Q, r) \quad (42)$$

(Hopp & Spearman 2008, 83)

According to Hopp, Spearman (2008, 83) there are two major challenges concerning total cost function of backorder approach, these challenges are estimations of variable A and b in reality. Unfortunately, Hopp & Spearman (2008, 83) states that backorder cost is very difficult to estimate in practice. But on the contrary, the primary aim is not really reducing the function

according to constraint but find the tradeoff between inventory average cost, service and set up cost. The next challenge concerning the equation is that the equations $B(Q, r)$ (36) and $I(Q, r)$ (41) assumes computation of Q and r variables in a difficult manner and using exact precise definitions is not possible. (Hopp & Spearman 2008, 83) However, approximation about $B(Q, r)$ (39) that has been done previously is very useful at that point, since it could be used to replace it's complicated analog also in the equation $I(Q, r)$ (41). Taking into account approximation the new functions is:

$$\begin{aligned}
 Y(Q, r) &\approx Y^{\sim}(Q, r) \\
 &= \frac{D}{Q} \times A + b \times B(r) \\
 &\quad + h \times \left[\frac{Q+1}{2} + r - \theta + B(r) \right]
 \end{aligned} \tag{43}$$

(Hopp & Spearman 2008, 84)

Then the values of Q and r that reduce $Y^{\sim}(Q, r)$ are computed as follows: Setting the resultative to zero and differentiating $Y^{\sim}(Q, r)$ according to minimum value of Q firstly:

$$\frac{d Y^{\sim}(Q, r)}{d Q} = \frac{-D \times A}{Q^2} + \frac{h}{2} = 0 \tag{44}$$

(Hopp & Spearman 2008, 84)

Then differentiating $Y^{\sim}(Q, r)$ according to minimum r and establishing result that is equal to zero:

$$\frac{d Y^{\sim}(Q, r)}{d r} = (b + h) \times \frac{d B(r)}{d r} + h = 0 \tag{45}$$

(Hopp & Spearman 2008, 84)

Further, it is sensible to take the equation of $B(r)$ (20) from base stock level as an approximation that substitutes $B(Q, r)$ (36) and compute the derivative of $B(r)$ (20):

$$\begin{aligned}\frac{d B(r)}{d r} &= \frac{d}{d r} \int_r^{\infty} (x-r) g(x) dx \\ &= - \int_r^{\infty} g(x) dx = - [1 - G(r)]\end{aligned}\quad (46)$$

The formula is obtained from (Hopp & Spearman 2008, 84) but for the complete derivation see (Zipkin 2000).

then the result of equation (45) is added to the result of equation (46) and summing it up:

$$-(b+h) \times [1 - G(r)] + h = 0 \quad (47)$$

(Hopp & Spearman 2008, 84)

Further, to reduce $Y^{\sim}(Q, r)$ the equations (45) and (47) has to be solved, it will be implemented through the formulas that have been derived in EOQ quantity and base stock level model concerning the optimal values of Q^* and r^* .(Hopp & Spearman 2008, 84)

$$Q^* = \sqrt{\frac{2 \times A \times D}{h}} \quad (48)$$

and r:

$$G(r^*) = \frac{b}{b+h} \quad (49)$$

At that point, an important observation is that approximation that has been done concerning base stock backorder level is resulting in a formula for r^* that has been derived in the base stock level model. Another notice is that Q^* is computed through the EOQ formula. (Hopp & Spearman 2008, 85) Furthermore, if demand during lead time could be modeled with normal distribution with mean θ and standard deviation σ , then the equation for r could be redone analogically to base stock level model:

$$r^* = \theta + z \times \sigma \quad (50)$$

where z is a value obtained from standard normal table. (Hopp & Spearman 2008, 85)

According to Hopp & Spearman (2008, 85), the values of Q^* and r^* are not precise actual values that correspond to reality. For validation, it is sensible to check them against order frequency, fill rate, backorder level, and average inventory on hand through the formulas (31), (34), (38) and (41). In case if the results do not correlate with reality, it is advisable to adjust the values of costs, particularly A fixed ordering cost and b backorder cost, as they are the most challenging to estimate. An important observation is that Q^* directly dependent on A , while average ordering frequency is inversely dependent on A . On the other hand, b directly influences r^* , but inversely decreases average backorder level and stock out rate. (Hopp & Spearman 2008, 85)

Stock out cost approach.

According to Hopp & Spearman (2008, 85) stock out cost approach could be defined analogically to backorder cost approach, if previously we have defined the equations for fill rate $S(Q, r)$ (34) and average inventory level $I(Q, r)$ (41). Substituting the textual notations in the equation (30) by mathematical ones that were derived previously, the total cost formula become:

$$Y(Q, r) = \frac{D}{Q} \times A + k \times D \times [1 - S(Q, r)] + h \times I(Q, r) \quad (51)$$

(Hopp & Spearman 2008, 85)

The total cost function in stock out cost approach now attains the same parameters and complicated challenges concerning their specification. In the current case, the stock out cost k is absolutely similar to the backorder cost b in terms of difficulties in its specifications. In addition, $S(Q, r)$ (34) and $I(Q, r)$ (41) involves both values of Q and r that are complicated to calculate together since in will produce difficult equation. Analogically with backorder cost approach, it is sensible to simplify them using approximations that have been made before. (Hopp & Spearman 2008, 85) Similarly with backorder cost approach, the influence of Q on the performance measures $S(Q, r)$, $B(Q, r)$, $I(Q, r)$ is eliminated, and indeed, the optimal order quantity is computed in the same way as in EOQ (48). (Hopp & Spearman 2008, 85) Further, according to Hopp & Spearman (2008, 85) the formulation of $S(Q, r)$ in total cost stock out function has to be substituted with type 2 approximation (35) that has been derived earlier, and respectively the backorder $B(Q, r)$ by base stock approximation $B(Q, r)=B(r)$ (39). After the complex performance measures in total cost function are replaced with simpler approximations, the total cost function is:

$$\begin{aligned} Y(Q, r) &\approx \tilde{Y}(Q, r) \\ &= \frac{D}{Q} \times A + k \times D \times \frac{B(r)}{Q} \\ &\quad + h \times \left[\frac{Q+1}{2} + r - \theta + B(r) \right] \end{aligned} \quad (52)$$

(Hopp & Spearman 2008, 86)

Again, when the substitution is made the total cost is minimized according to the optimal reorder point r by deriving it with respect to r , establishing equality to zero and solving r , results in the following expression:

$$G(r^*) = \frac{k \times D}{k \times D + h \times Q} \quad (53)$$

(Hopp & Spearman 2008, 86)

In case if demand during replenishment lead time is possible to model with normal distribution (where: θ = mean, σ = standard deviation) then the equation (53) is expressed as:

$$r^* = \theta + z \times \sigma \quad (54)$$

Where $\Phi(z) = k \times D / (k \times D + h \times Q)$.(Hopp & Spearman 2008, 86)

Important observation is that equation (49) differs from the equation (53) that was derived in backorder cost approach in terms of its sensitivity to Q . Looking at the expression (53), it is obviously seen that increase in Q will decrease the right part of equation (53) and consequently also decrease reorder point r^* . The explanation of that is the fact that the average inventory level tends to increase with bigger values of Q and subsequently the fill rate is also increasing. Accordingly, it results in smaller reorder point that is needed to reach certain service level. (Hopp & Spearman 2008, 86)

Lead time variability in (Q, r) model.

The initial representation of (Q,r) model obtains the assumption that lead time of supply is known and fixed. Nevertheless, in reality, it very rare corresponds to the truth, since suppliers tend to be late with deliveries quite often. (Hopp & Spearman 2008, 88-89) Taking into account lead-time variability is a very important action concerning the inventory control, since

it may directly influence the level of stock outs. In order to replicate the essence of (Q, r) model more to reality, it is essential to account for lead time variability and develop its modeling equation with following assumptions:

L = lead time (days), a random variable

$l = E[L]$ = expected lead time (days)

σ_L = standard deviation of lead time

D_t = Demand occurring at a day t , random variable. It is presumed that demand is time independent and has the same distribution for each day. In addition, demands are assumed to be independent on each other

$d = E[D_t]$ = expected daily demand (units)

σ_D = standard deviation of daily demand (units)

(Hopp & Spearman 2008, 88-89)

Another fact that has to be stated is:

$$X = L \times D_t \quad (55)$$

where X is the demand during lead replenishment time, as it was in the previous version of (Q, r) model. (Hopp & Spearman 2008, 88-89)

It is assumed that demands occur independently day by day and moreover, they are similar. Accordingly, the expected demand during lead time is

$$E[X] = E[L]E[D_t] = l \times d = \theta \quad (56)$$

That was broadly used as θ in the previous version of (Q, r) and base stock level model. (Hopp & Spearman 2008, 88-89)

Hopp & Spearman (2008, 88-89) claim, that variability in lead time also affect the variability of demand during lead time and complete its' variability according to the formula for independent, similarly distributed random variables:

$$\begin{aligned} \text{Var}(X) &= E[L] \times \text{Var}(D_t) + E[D_t]^2 \times \text{Var}(L) \\ &= l \times \sigma_D^2 + d^2 \times \sigma_L^2 \end{aligned} \quad (57)$$

Accordingly, standard deviation of lead time demand is:

$$\sigma = \sqrt{\text{Var}(X)} = \sqrt{l \times \sigma_D^2 + d^2 \times \sigma_L^2} \quad (58)$$

(Hopp & Spearman 2008, 88-89)

Another option is the case when demand is Poisson. In that situation $\sigma_D = \sqrt{d}$, due to the fact that standard deviation represents the square root of the mean when taking into account random variables in the Poisson process.

(Hopp & Spearman 2008, 88-89) If it is replaced to the equation (58), then lead time demand is defined as:

$$\sigma = \sqrt{l \times d + d^2 \times \sigma_L^2} = \sqrt{\theta + d^2 \times \sigma_L^2} \quad (59)$$

Important observation that if $\sigma_L = 0$ then lead time demand is defined as $\sigma = \sqrt{\theta}$ what has been relevant in the case where the assumption was so that lead time is constant. (Hopp & Spearman 2008, 88-89)

Summing up the (Q, r) model.

The theoretical part of the (Q, r) model, specifically its mathematical definition and computational analysis are quite sophisticated, comparing to EOQ quantity model and base stock level model. However, the general mechanics of the (Q, r) model is similar to the one that is used in the base stock level model and EOQ. Particularly, 1) Cycle stock is increasing while the frequency of replenishment is decreasing. 2) Safety stock is a buffer against stock outs, and as SS is increasing, the reorder point is also increasing. (Hopp & Spearman 2008, 90)

By the light of a nature, the (Q, r) model provides a rough understanding of the fact, that service level, safety stock, backorder level are mostly influenced by the position of reorder point r . Analogically, cycle stock and ordering frequency are directly influenced by order quantity Q . (Hopp; Spearman 2008, 90)

Nevertheless, mathematical computation of the model provides the true mechanics of the model in terms of the influence of both variables, Q , and r on the fill rate and backorder level. The general essence of qualitative observation concerning the (Q, r) model is so: the bigger is the optimal order quantity Q , the less is the frequency of replenishment orders, the rarer reorder point is reached, and the likelihood of stock out situation is smaller. On the contrary: if the replenishment quantity Q is small, it results in higher replenishment frequency, more often crossing of inventory position through reorder point and higher stock out's likelihood. (Hopp & Spearman 2008, 90) In addition, the basic observations obtained from the resulting formulas (48), (50) (54) could be stated as:

- 1) The rise in the value of average annual demand D tends to rise the value of Q .
- 2) The variability in demand during lead-time θ tends to increase the position of reorder point r . Additionally, the rise in annual demand D or the rise of lead time L tends to increase the demand during lead time θ . The explanation of this is the fact that high demand and long lead time require higher inventory level to buffer against uncertainty.
- 3) The standard deviation of demand σ is directly influencing the reorder point and the amount of safety stock. The explanation of this fact is the notion, which higher external uncertainty and instability will result in higher likelihood of stock out and hence, the higher requirement of safety stock.
- 4) The holding cost h is possible to increase in two ways: to increase the value cost of an item or to increase the interest rate of inventory. Hence, the rise in holding cost tend to decrease the value of Q and r . It could be explained in a way that higher costs of average inventory holding are motivating to keep less inventory.
- 5) Fixed ordering cost A , backorder cost b , stock out cost k is very difficult to estimate in reality to get precise actual values. These cost values are very sensitive in terms of their influence on the model. If

after the model formulation, the result does not seem to be adequate or performance measures $S(Q,r)$, etc. does not correspond to reality, it is sensible to adjust the A , b , k costs.

(Hopp & Spearman 2008, 90)

3.5 Mathematical Modeling

What is mathematical modelling?

Mathematical modeling is a technique used to describe or predict the behavior of real world systems with mathematical functions and numbers. (Lawson & Marion 2008, 1) Mathematical modeling is broadly used in Operation research problems and particularly in problems, which are working with certain decision variables. Generally, this process starts from the formulation of the problem verbally, where the researcher defines the fundamental aspects of operation research models: the decision variable, objectives and constraints, which will be then modeled or formulated in the mathematical equation or function. Afterwards, this equation is solved according to several scenarios and results obtained from it are analyzed. Important to notice that the results have to be concluded from the mathematical model, but not from the problem's feature that is represented by it. Accordingly, the results should be then analyzed critically and tested against interference, which claims that they are meaningful and could not be applied to the real-world problems or features that model is intended to describe. (Rardin 2017, 3-4)

The reason for it is that real world systems are often extremely sophisticated and complex groups of different objects, which are diversified in a variety of their features so that their simulation and modeling with mathematical tools are either very difficult to initiate or it will require applying advanced mathematical knowledge and skills. Subsequently, there is a number of

important points that have to be considered before the formulation of the problem. First, the important point is the fact that natural events and systems are often very complicated in their structure. Generally, such systems include a lot of different functions, members, and objects, whose behavior usually does not have a crucial effect on the modeling but could make it far more complicated to replicate it by numbers. Consequently, it is very important to identify such parts of the system and exclude them from a modeling perspective. The second remark is related to the amount of mathematical processing that should be included in the model. Empirically, mathematics has proven its' robustness through significantly long time-period by providing useful and optimal results in problem-solving. However, the results obtained from mathematical calculations are extremely dependent on the equations and mathematical laws that are implemented in it. Insignificant changes in the equation may require significant changes in the mathematical model. Another crucial factor at that point is a number of variables used in the equation. The higher number of variables increases the level of uncertainty dramatically and hence; it is more difficult to simulate and obtain proper results. (Lawson & Marion 2008, 1)

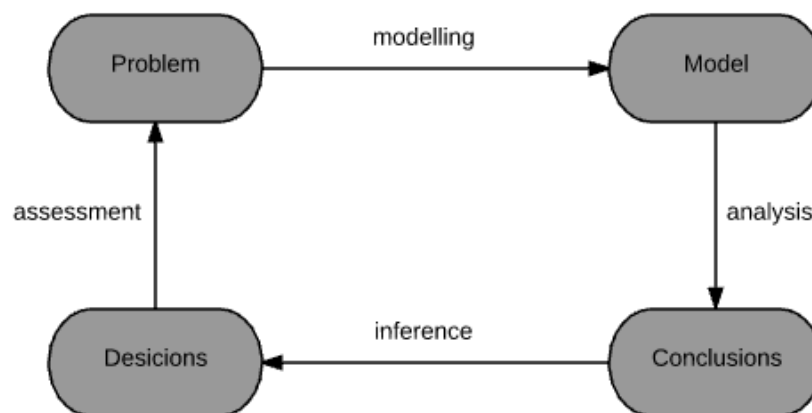


Figure 19. Operations Research Process by Rardin (2017, 3)

The key three dimensions of any modeling problem are decision variables, constraints, and objectives. The proper definition of these three aspects is a must, which is needed to clarify the problematic features and questions of the study. (Rardin 2017, 3-4)

The definition of the decision variable is a straight-forward according to the Rardin, (2017, 3-4) Usually, this term is referred to the quantifiable solution that answers traditional questions of problems with uncertainty. In the scope of inventory systems research, the example of the decision variable could be the optimal order quantity which is need for economical operations. (Rardin 2017, 3-4)

The term constraint describes modeling feature, which restricts the number of possible solutions. The example of constraint in inventory system problem could be represented as service level that is established by an enterprise in order to reach a certain level of performance. (Rardin 2017, 3-4)

Finally, the last element is objective. The objective is referred to the solution that tends to be the most beneficial among the number of all other possible solutions. At the same time, it has to be capable to satisfy the constraints. In the area of inventory systems, the example of the objective could be the reorder point that minimizes total cost of inventory. (Rardin 2017, 3-4)

3.5.1 Optimization

According to the Snyman (2005, 19), numerical optimization is a science, which tends to determine the best solution among the mass of all possible solutions in problems with uncertainty. The best solution is usually referred to the definition of the maximum or minimum result in the problems where the decision has to be made under suspense, and the variables can randomly take values over the certain numerical range. The standard optimization

problem consists of the function $F(X)$, which output has to be maximized or minimized. In addition, it includes variables X_1, X_2, X_3 , etc., which could take values over certain numerical range and constraints, which establish the limits for the range of potential decisions. (Snyman 2005, 19) The example of Optimization problem could be demonstrated on traditional inventory problem where: q = reorder quantity, which is purchased periodically and r = reorder point that generates the signal for new replenishment batch. The constraints than are defined as:

$$q \geq 100$$

$$r \geq 0$$

The objective function that is supposed to be minimized is:

$$c(q, r) = \text{total cost with order quantity } q \text{ and reorder point } r$$

Consequently, the optimal solution in this case is when the total cost function is minimized with respect to the constraints $q \geq 100$ and $r \geq 0$. (Rardin 2017, 5)

3.5.2 Sensitivity analysis

Sensitivity analysis is a mathematical tool that is used to determine the sensitivity of the system's output by establishing the change in the input parameter or makeup of that system. Usually, the change is initiated in the one of the following: 1) the input parameters are slightly changed from the initial value 2) the system makeup including system parameters, initial and boundary conditions, etc. are changed. Apart from the What if? analysis, which deals with any kind of change, the sensitivity analysis considers small changes in the system makeup or input parameters and demonstrates how this insignificant change influences the output value. (Carmichael 2013, 375) According to Carmichael (2013, 375), the system is considered to be highly-sensitive when the small change in the system's input parameter or makeup generates a large change in the output results that system produces. On the other hand, the system is considered to be low-sensitive, when the small

change in the system's input parameter or makeup generates insignificant change in the output result of the system.

In order to distinguish the impact of the change in other input parameters, it is recommended to change only one input parameter at a time and keep the rest of the parameters constant. In this way, the sensitivity analysis could be initiated for all the input parameters of the system with respect to the change in the output. (Carmichael 2013, 375)

4. Research implementation

4.1 Process development of the model

The definition of process development is an important stage when creating any kind of an operational model. The aim of this procedure is to provide the user with step-by-step information about model formulation from the conceptual stage to the stage of validation. The availability of that information is useful in a sense that it will visualize the detailed procedure of the formulation and help to find a root cause of the potential mistakes and flaws in the model that possibly could have been made during the development process. The author adopted the framework of the process development definition from Tersine (1994, 591) and modified it according to the core of the model and research. Tersine's seven steps are: problem definition, construction of a conceptual model, collection of data, development of the model, model validation, experimentation and analysis of the results.

However, even though Tersine's (1994, 591) steps are a complementary structure of the modelling development process, they are quite general, and depending on the situation, they might be slightly customized according to the case. For example, the present study assumed the development of the statistical inventory model. Particularly, it means that the model will not

obtain any dynamic properties but, instead, it will include a number of calculators and databases, which will allow to compute the calculation of the statistical inventory values and show the accumulated statistical inventory data. Furthermore, no simulation or inventory tracking options will be provided. Subsequently, the “Experimentation” stage is not possible to perform due to the lack of dynamic features. In addition, the author changed the order of the process steps according to his own preferences, convenience and vision about the process. The modified process development structure is:

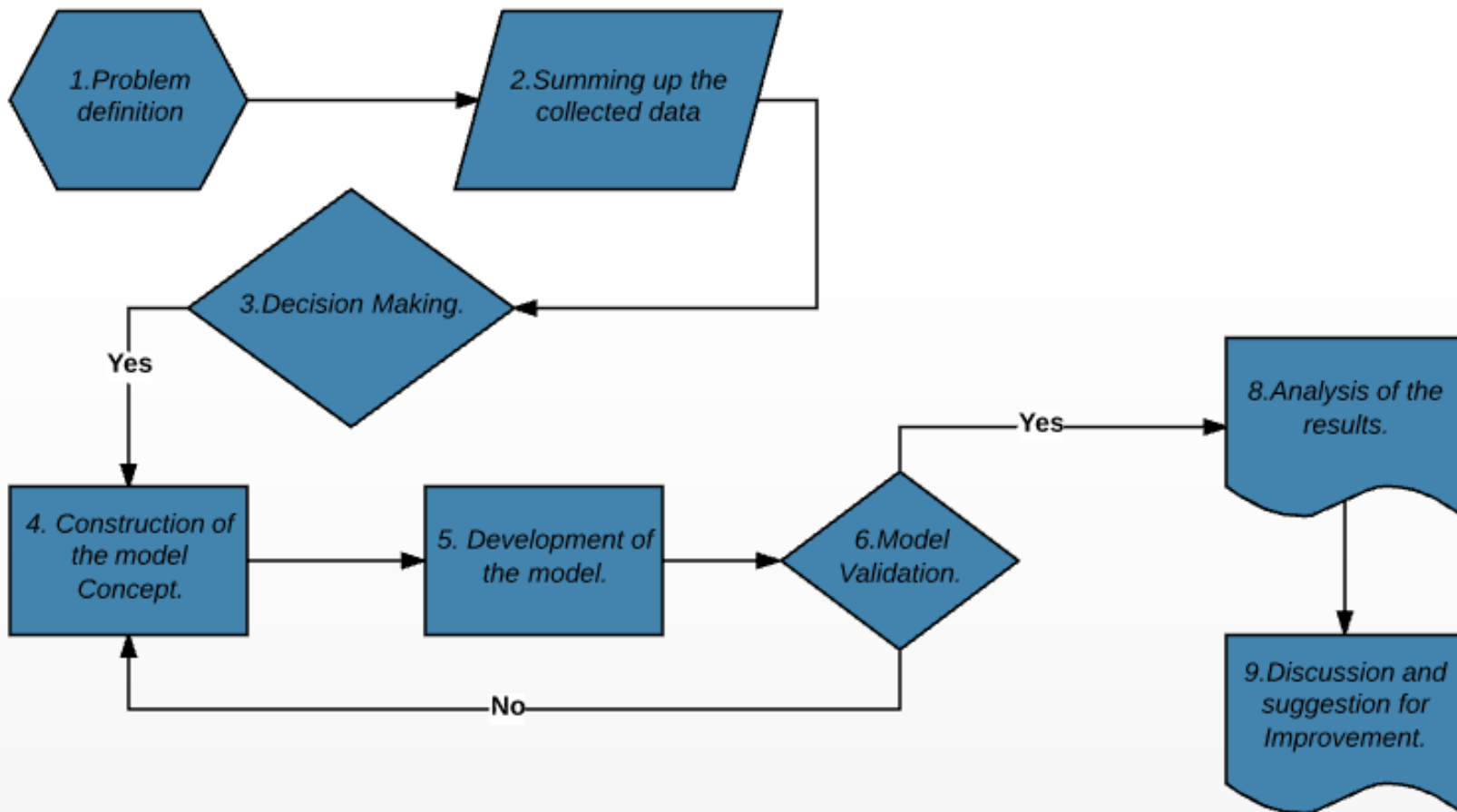


Figure 20 Development process of the model.

4.2 Problem definition

The primary aim of the study was to examine and analyze different materials management and inventory control approaches that could be implemented to the BB Inc. case. Consequently, based on the obtained data, the author had to develop a new inventory model that would efficiently support the manufacturing operations of BB Inc. The strategic purpose of the model was set by Mr. Seppo Koiranen, the author's supervisor from BB Inc. and it states: *"the inventory model should provide the required level of components' availability with respect to the established service level and at the same time control the expenses related to the stock so that the inventory costs would stay at the minimal level."*

The challenging point of the research process was the fact that at the stage of new product development, the organization had not obtained any precise information about the demand for the products and components.

Particularly, in the given case no specific data and requirements were available due to the fact that before the implementation of this study, BB Inc. had not applied any professional materials management approaches to their operations. However, the only information that was clearly provided to the author was the approximate demand scenario of the product created by the project team, the variable lead time that was specified by the supplier and the business model of the organization that corresponded to the Make-To-Stock system. Furthermore, the author's supervisor from the BB Inc., Mr. Seppo Koiranen, specified that the R&D department of the BB Inc. had not had any experience in the implementation of MM approaches into the organizational structure, and these methodologies were absolutely new to the enterprise. Accordingly, they were not able to guide the author with professional advice based on experience. On the other hand, the organization was willing to provide the author with the opportunity to apply

his ideas, knowledge and experience to this project, but only after a discussion and agreement with the project team and management of the company. In addition, the management of BB Inc. denoted that the improvement of the company's performance through costs reduction would be a crucial strategic movement towards operational excellence. Accordingly, they were willing to know the costs of the inventory model under development and, moreover, they were also interested whether it was possible to minimize the costs by means of optimization or any other relevant method.

4.3 Summation of the collected data

The process of data collection started from the project meeting where the author was introduced to the management of BB Inc. and, particularly, to the members involved in the On-Demand control system's development project. This meeting was the starting point for the whole project, and all the necessary requirements, sub-projects, schedules and deadlines were defined at that point as well the initial requirements concerning the thesis research and other relevant assignments related to the On-Demand Control System development. Furthermore, in order to be able to participate in the project, the researcher had to form a good understanding of the On-Demand control system by reading the technical drawings, circuit diagrams and other engineering specifications. Particularly, this included electrical components' circuit diagrams, hardware, software, specific valve configurations based on the flow pressure, a hydrostatic braking system, a protective case, spare parts and other specific items. Although the definition of the bill of materials was not included in this thesis, the author had to do it in order to be able to analyze the structure of the product and understand the dependence of the components or the variance in the system's configurations. The necessity of

this step was a must, since it had an effect on decisions on, for example, whether the inventory systems should be based on the independent or dependent demand pattern or whether the inventory system requires a multiproduct or single product solution. In addition, the author had been observing the materials management operations at the enterprise, as well as the manufacturing operations at BB Inc. The substantial understanding of the organizational and MM structure in BB Inc. was received through the observation processes and interviews with the company members, particularly: Mr. Seppo Koiranen, the Technical Director, Mr. Jakke Palonen, the Laboratory Manager and simultaneously with the main assistant of Mr. Koiranen in the On-Demand Control System project, Mr. Raimo Piippa, the Supply Chain Manager. Nevertheless, the most significant part of the data for the research was obtained from different literature sources including books, articles and scientific studies on industrial engineering, open studies provided by the supply chain consulting companies and other sources. The crucial information concerning inventory models and their mathematical mechanisms was adopted from the "Factory Physics" approaches developed by Hopp & Spearman (2008). Particularly, three approaches of inventory models' development (EOQ, Base stock level model, the (Q, r) model) were studied and compared between each other and the organizational structure of BB Inc. Consequently, according to the coincidence between the theoretical inventory solution and the case of BB Inc. the decision was made.

Table 5 Classification of Inventory Models and Coincidence with BB Inc.

Modelling feature	EOQ	BSL Model	(Q, r) Model	Black Bruin Inc.
Continuous (C) or Discrete (D) time	C	C	C	C
Single (S) or Multiple (M) Products	S	S	S & M	S & M
Backorder (B) or Lost Sales (S)	—	B	B	B
Fixed Ordering Cost Yes (Y) or No (N)	Y	N	Y	Y
Deterministic (D) or Random (R) Demand	D	R	R	R
Defines order quantity (Q) or reorder point (R)	Q	R	Q & R	Q & R
Deterministic (D) or Random (R) production	D	D	D	D
Finite (F) or infinite (I) Horizon	I	I	I	I
Fixed (F) or variable (V) lead time	F	F	V	V
Finite or Infinite production rate	I	I	I	I

The information obtained from the research and literature review has been analyzed and summarized in Table 5, where the most relevant modelling features or assumptions are presented. In the classification table, it is clearly seen that the (Q, r) model corresponds to the case of BB Inc. in the most eligible manner according to its relevance and applicability. The most important reasons it was decided to develop the (Q, r) model were:

- 1) Apart from the EOQ and Base stock level model, the (Q, r) model allows the computation of both decision variables: the optimal order quantity and reorder point. Moreover, the (Q, r) model solution presented by Hopp & Spearman (2008) is a combination of mathematical mechanisms from the EOQ model and Base stock level model. In addition, the (Q, r) model allows to implement the approximation techniques concerning mathematical formulas and, therefore, it could be constructed in an excel even though it contains an integral calculation.
- 2) Apart from EOQ, the (Q, r) model accounts for the uncertainty of the demand.
- 3) The (Q, r) model takes into account the variability of the lead time.
- 4) The products under the (Q, r) model are possible to analyze according to a multilevel or individual level approach.
- 5) The mathematical mechanism of the (Q, r) model is relatively demanding, but its' complexity results in relatively reliable and precise

outputs. Furthermore, the model allows a certain number of possible improvement techniques that might increase its robustness and accuracy. The author also considered the fact that potential development could be implemented to the model in future if it is required.

4.4 Construction of the model concept

The initial aim of the constructive model was to provide a tool that would support BB Inc. in the decision-making process regarding their inventory operations. Firstly, it was assumed that the basic mechanism of the model would automatically compute the calculation of two main decision variables: order quantity Q and reorder point r . Consequently, when the main decision variables are computed, the rest of the relevant stock components and performance measures, such as the base stock level, safety stock, fill rate, average inventory on-hand etc. could be computed with respect to the values obtained from the first calculation. The secondary aim of the model was to create the costing model of the constructed inventory system and specify the dependency of the costs and their relationship with regard to the performance measures and stock components. Therefore, the overall purpose of the inventory model was to increase the visibility of the stock control operations, provide the tool for the inventory costs analysis and also define the correlation between performance measures, stock components, and the main cost drivers. This visibility would be a must for the inventory control of BB Inc. since it would increase the level of understanding regarding the basic mechanics of inventory and also it would help to define potential methods to improve its operational efficiency.

First of all, it was assumed that the model would be created in the Excel spreadsheets since it is an efficient and relatively simple tool to use.

Moreover, it includes an extended package of mathematical solutions, and it is available on almost every personal computer.

At the next stage of conceptual development, the author assumed that the model would have one main display with a calculator and the second one with a database, where the information of the costs and parameters of modeling assumptions would be entered. The general idea of the algorithm was so that the user would create the item, enter the variable for the costs and modeling parameters, such as lead time or demand of the product into the database. Further, the user would proceed to the second display with the model calculator where he would be able to compute the values of Q and r by pressing the button “to compute the values”.

Press here to compute the variables											
Item number	Optimal Order Quantity Q	Base stock level	Safety stock	Reorder point	Ordering frequency $F(Q)$	Fill rate $S(Q, r)$	Average Backorder level $B(Q, r)$	Average Inventory Level $I(Q, r)$	Inventory turnover ratio	Inventory through put time	Inventory Holding cost
1	4	7	1,27397	3	3,5	97,4 %	0,105650112	3,879622714			116,39 €

Figure 21 Initial user interface design of backorder's calculator.

At this point, the author made an important consideration that it would be advisable to create a functional button, which would allow to increase or decrease the values of Q and r in anticipation with formulas or when the calculation was completed. After the values are changed, the user is able to complete the calculation of the performance measures again with new adjusted values of Q and r . This option is useful in the sense that it would allow the inventory manager of BB Inc. to make some sort of what-if scenarios by changing the decision variables and see how it will influence the fill rate and Inventory holding cost. For example, if after the first calculation, the inventory manager decides that the fill rate of **97.4%** is too low, there are two options: to increase it by changing either value of Q or value of r . Actually, increase in both of the values will increase the fill rate, but on the other hand, it will affect the total cost of inventory in another way as well as

the average level of backorders.

The next complement to the model was raised by the necessity to analyze the statistical inventory data computed from the calculator. This option would be useful since it would allow the inventory manager to analyze the total costs of inventory, which were cumulated from individual items as well as other performance measures on an average or total basis. The opportunity to analyze computed statistical data from the aggregate perspective would provide a possibility to increase operational efficiency even more, as the common picture might shed light on the new opportunities to reduce the costs or just simply would help to find the potential mistakes or wrong decisions that possibly could have been made during the previous calculations. The availability of a database for the calculated items would also allow visualizing the situation by providing the graph with average or total values of performance measures and costs.

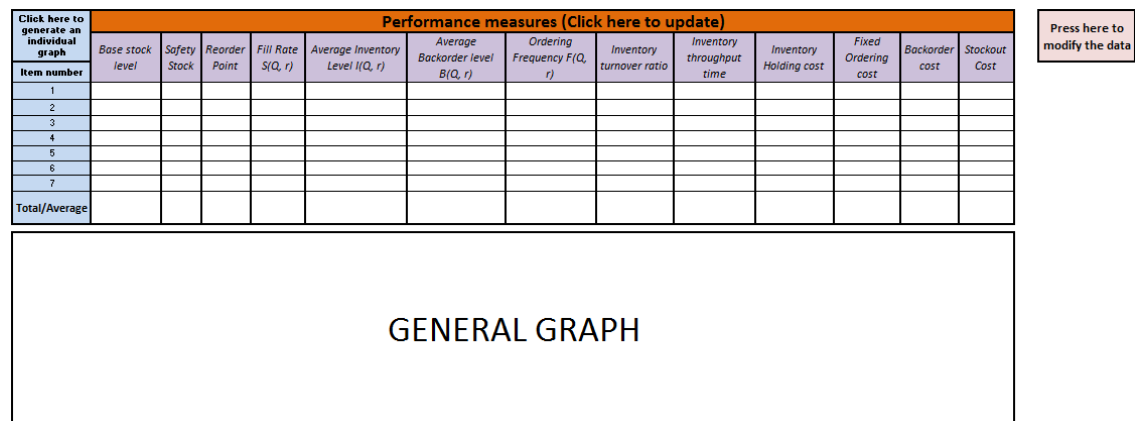


Figure 22. Initial User Interface Design of database for calculated items.

In the next step, when the researcher gathered the information about inventory costs and methods for their calculation, he realized that the actual inventory costs were going much deeper than just the holding cost, ordering cost and cost of stock out or backorder. Each of the costs usually has several components that constitute it. In addition, it is advisable to calculate some of

the components by using other factors, such as interest rates or WACC (weighted average cost of capital). Accordingly, at this point the researcher decided to create one more specific calculator for the costs, which would allow calculating the costs of the inventory for each individual item taking into account their internal components, interest rates and WACC.

Press Here to Add/Change parameters for WACC											WACC	
Weight of debt	Weight of Equity	After Tax Cost of Debt	Corporate Tax Rate	Cost of Debt	Cost of Equity	Adjusted Risk Free rate	Annual Inflation Rate	Country risk premium	Risk Free Rate	Unlevered Beta	Market Premium	10,24 %
-6,24 %	106,24 %	1,12 %	20 %	1,40 %	9,70 %	2,90 %	1,50 %	0,50 %	0,90 %	1	7,10 %	

Press here to add the variables for the costs													
Item number	Costs		Ordering cost		Total	Holding Cost				Total	Backorder cost	Stockout cost	Total
	Influencing factors	The process	Inbound logistics	Capital		Space	Insurance	Risk	Estimation		Estimation		
	Interest	1 %	0,5 %	5,0 %		0,5 %	0,5 %	Estimation	Estimation				
	Value												
1	1.000,00 €	10,00 €	5,00 €	15,00 €	102,35 €	50,00 €	5,00 €	5,00 €	162,35 €	200,00 €	40,00 €	177,35 €	
2	- €	- €	- €	- €	- €	- €	- €	- €	- €	100,00 €	40,00 €	- €	
3	- €	- €	- €	- €	- €	- €	- €	- €	- €	100,00 €	40,00 €	- €	
4	- €	- €	- €	- €	- €	- €	- €	- €	- €	100,00 €	40,00 €	- €	
5	- €	- €	- €	- €	- €	- €	- €	- €	- €	100,00 €	40,00 €	- €	

Figure 23 Initial User Interface Design of costing calculator.

Further, the last addition to the conceptual version of the model was to include one more calculator for the decision variables. Since the (Q, r) model allows calculation of Q and R on the basis of the critical fractile obtained either from the stock out cost or from the backorder cost approaches, it would be sensible to implement both of them, as different methods produce different values. Consequently, the author decided that a higher variability of options for calculation would provide a better understanding of the inventory mechanics and control. In addition, based on the inventory policy and chosen approach, BB Inc. would be able to use both of the calculation methods. The interface of the stock out calculator was designed in the same way as the interface of the backorder calculator, and the only difference was the mathematical formulas inside.

Finally, when all preliminary options and parameters of the model had been identified and agreed with the management of BB Inc. as well as with the author's supervisor Mr. Sipilä, the author was able to define the preliminary functional algorithm and start the development of the model into the Excel

spreadsheet file.

The algorithm is shown in Figure 24.

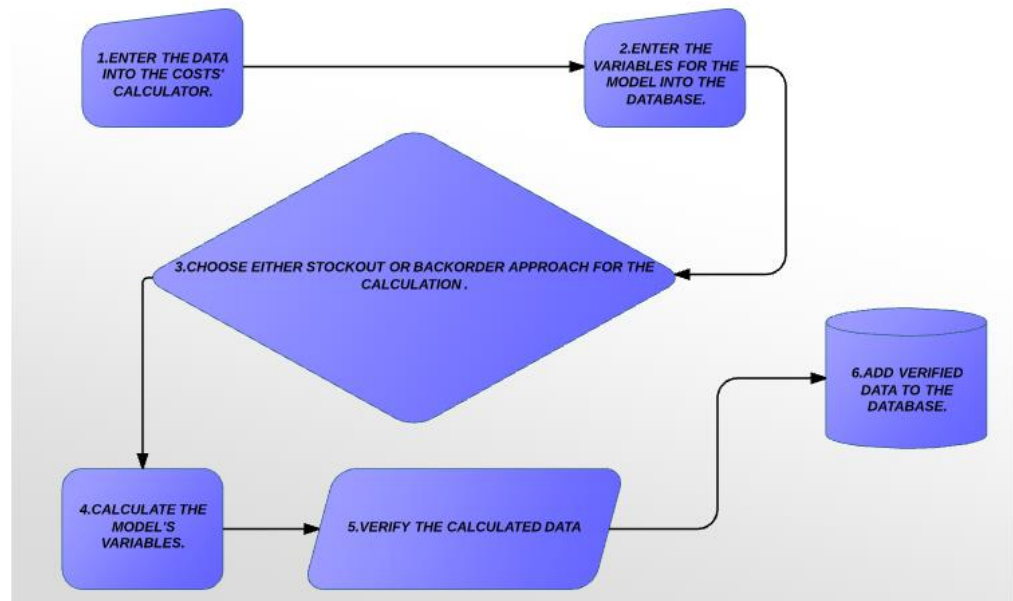


Figure 24 Preliminary algorithm of the model's usage.

4.5 Development of the model

This chapter describes the specific features related to the development of the model. As it was mentioned in chapter 4.3, the solution was to develop the (Q, r) model, which is represented by Hopp & Spearman (2008). Consequently, all mathematical mechanisms of the model were completely adapted from Hopp & Spearman (2008) Factory Physics Inc. Accordingly, the explanation of each modeling feature is not provided in this chapter, since all the mathematics of the model could be found in Chapters 3.3.4-3.4. Nevertheless, a specific modeling decision and assumptions related to the development of the model are explained and analyzed.

4.5.1 Why VBA?

As the main functions of the model were to compute the calculation of the decision variables and indicate the statistical data for the inventory, the Excel spreadsheets apparently turned out to be the best solution for it. However, due to the fact that the model includes around 60 mathematical formulas, and some of them require advanced knowledge and understanding of mathematics and statistics, it might be too difficult to use the model for those who do not have this knowledge and experience. Thus, the author decided to develop the model so that the users do not have to operate with the mathematical mechanism directly. Instead, they can insert the input variables or parameters into certain cells and complete calculations automatically. Consequently, the author developed the idea of applying the capabilities of VBA (Visual Basic for Applications) in the Excel spreadsheets. The Visual Basic for Applications is a standard computer programming language, which is included into the basic Microsoft Office package. VBA allows creating specific functions or features in the Excel, Word or PowerPoint packages according to the preferences and requirements of the user. Particularly, VBA in the Excel is useful when processes or calculations have to be automated and completed with one single click rather than copied and computed manually. The main advantages of using VBA in the model are: 1) It accelerates the process of calculations and the transitions of the calculated data from one spreadsheet to another. 2) It decreases the possibility of an error during the transition of data or the process of calculation since VBA does not imply the profound understanding of the sophisticated model's structure in spreadsheet tables. 3) The implementation of the Visual Basic code will provide the possibility to use the model even for those who do not have deep and advanced knowledge in mathematics and inventory models. Therefore, the use of the model will not imply long education and training for the BB Inc. management, but it will only require providing them with the manual-algorithm of the model's usage

and information about the basic performance measures, variables, their definition and how they are correlated with each other.

4.5.2 User Interface Design

User interface design is an important aspect of any functional application. Based on the needs of the company, the UI of the model should be structured, easily understandable and it should not confront with the visual perceptions of the user. Accordingly, the disposition of the control buttons, the selection of the colors for the control buttons and important cells as well as the size of the font should blend harmoniously and influence on the visual cognition of the user in a positive way, reducing overload of the user's sensory perception. Furthermore, the algorithm of the model should also be consistent and straight-forward, so that the user could easily navigate between spreadsheets and the phases of calculations.

4.5.3 Estimation of the input variables for the costs

In spite of the fact that the mathematical mechanism of the model was completely obtained from Hopp & Spearman (2008), some of the input variables were estimated. Particularly, this issue was related to the estimation of the interest rate factors for the costing components and costs itself. According to the specificity of the management accounting disciplines in Black Bruin Inc, it was decided to model the costs of inventory on the basis of an item's value. Therefore, the total cost of the inventory would be calculated for each item separately on an annual basis and then summed together to display the total cost of the inventory for all items. With respect to the method, the costs were divided into components with a certain factor of interest rate that corresponded to each costing component. Particularly, the holding cost of inventory was divided into four components: the cost of capital, cost of the facility, inventory service cost and cost of the risk. All

costing components, except the cost of capital, were calculated according to the interest rate factor that was established by the management of the company. This interest rate factor is represented as a percentage share of the total value of an item.

Apart from the rest of costing components, the cost of capital was calculated with the WACC method. For this particular case, the method was initiated as follows:

$$\text{Cost of Capital} = \text{Value of item} * \text{WACC} \quad (11)$$

The computation of the WACC value is a rather complicated procedure, and for its precise definition, it would be sensible to implement another study, which would aim to identify and analyze the drivers behind the WACC factor in the case of BB Inc. However, as the author did not have enough resources or theoretical knowledge to perform additional research, and as the management accounting methods of Black Bruin Inc. had never dealt with the usage of WACC, the author decided to use the statistical value of WACC in the technology-sector of Finland. The WACC value was obtained from the financial consulting organization **WACC Experts (2017)**.

Analogically with the Inventory holding cost, the ordering cost was also divided into two components: the cost of the clerical process and the cost of the inbound logistic process. However, in this case, the method of setting a particular interest rate was irrelevant, since in reality the ordering cost was not influenced by the actual value of an item. In accordance with this, it was decided that for both components of the ordering cost a particular value would be established, which would correspond to the amount of money related to the performance of the operations behind the costing component. The responsibility to establish this value fell to the BB Inc. management.

However, the most crucial issue related to the modeling of the inventory costs was dealing with the definition of the backorder cost and stock out cost. According to many experts, including Hopp & Spearman (2008), Harris (1913) etc., the definition of these costs, in reality, is a challenging task, since the drivers behind them are intangible measures, and their estimation in real cases is almost impossible to initiate precisely. The BB Inc. case was not an exception, as the financial operators had never taken into account such phenomena as backorder or stock out cost. In addition, the capabilities of the author to determine these factors in a sophisticated scientific way was very limited as well as the resources for it. Subsequently, the backorder cost and stock out cost were completely estimated for this case. Nevertheless, both costs had considerable influence on the final result, and their estimation must have had at least some sort of solid basis lying beyond them. Consequently, it was decided to implement a sensitivity analysis of the backorder and stock out cost and define the numerical range where the values of these factors would produce more or less adequate results. Specifically, according to Hopp & Spearman (2008, 78), the value of the backorder or stock out cost could be considered acceptable if it generated a fill rate higher than 90% percent. Any value of the fill rate lower than 90% may have indicated that the choice of the value for the backorder or stock out cost was too low.

4.5.4 Optimization

The current research also implied the optimization of the total inventory cost. In mathematics, the basic idea of optimization refers to the derivation of the minimum or maximum value of the certain function. In the case of Black Bruin Inc., this function was the total cost of inventory that was expressed in equation (43) for the backorder cost approach and in equation (52) for the stock out cost approach. For this particular case, the cost function had to be minimal, and at the same time, it had to satisfy the

constraints. In this situation, the constraint was represented as the service level value of ***%, which was set by the management of the organization. All in all, the optimization of the cost function was implemented through the differentiation of a total cost equation for the backorder (43) approach and stock out approach (52) with respect to the value of Q and r and establishing the result to be equal to zero.

With the backorder cost approach, the equations for the optimal order quantity and the optimal reorder point were derived from equation (43) and represented in equations (48) and (49). For the stock out cost approach, the optimal order quantity and the optimal reorder point were derived from the equation (52) and expressed in equations (53) and (54).

4.5.5 Sensitivity Analysis

A sensitivity analysis is necessary to implement in almost all cases when a decision has to be made under uncertainty. In the given case, the extent of uncertainty was significant, since some of the input parameters were challenging to estimate. In this particular case, this statement was relevant for the backorder and stock out costs. Although the precise estimation of them was rather complicated, their significance for the final result was crucial. Despite the fact that the definition of those values was referred to the responsibility of BB Inc. management, it would be sensible to implement a sensitivity analysis and identify the domain of values, which would produce the most qualified and genuine outputs. The accuracy of the backorder and stock out cost was mostly relevant for the computation of the fill rate. As stated by Hopp & Spearman (2008, 78), a too low service level most certainly means that the value set for the stock out cost or backorder cost was too low. Accordingly, the primary aim of the sensitivity analysis in this research was to identify the sensitivity of the service level with respect to the change in the value of the backorder or stock out cost. However, the service level was computed by taking into account the expected average level of the

backorders, while the expected average level of the backorders was computed on the basis of the critical fractile that was represented in formula (49). The value of the critical fractile is different for each individual item, since it is computed through the relationship expressed in equation (49), where the value of the holding cost varies according to the value of the item. Almost the same dependency could be seen in the stock out cost approach. Accordingly, in order to generate an adequate value of the service level for each individual item, a sensitivity analysis had to be respectively performed for each calculation, since each item obtained different input parameters. However, the potential users would not be able to perform a sensitivity analysis in the process of using the model, as they do not usually have the proper knowledge concerning the mathematics of the model. Therefore, it was decided to include a sensitivity analysis into the model so that it would highlight the values of the backorder and stock out cost with green, yellow and red color. The green color would mean that the value of the cost was appropriately set for the calculation, while the yellow color would mean that the value of the cost was slightly underrated, and the red color corresponded to a significantly underrated value of the backorder or stock out cost. The results of the sensitivity analysis are presented in Chapter 5.1.4.

4.6 Model Validation

Finally, when the model was constructed, it was necessary to validate it. Validation was required to ensure that the model would correspond to all the requirements set by the author and that it would generate adequate outputs. In the particular case, when the results were computed by the calculators with the formulas inside, validation was needed to ensure that these formulas were functioning correctly. One way to validate this issue was to enter the input variables from the existing case-calculation into the model

and check whether the obtained results corresponded to the results from the case-sample. At this point, it was important to notice what kind of modeling techniques were used in the case sample calculation and whether the same modeling assumptions were implemented into the particular model. If the modeling assumptions were coincident with the assumptions defined in the case-sample and at the same time, the obtained results were identical, the mathematical mechanism of the model could be considered correctly implemented. Therefore, the model was considered to be authentic and robust.

During the actual research process, the model was tested several times with different input parameters from case-calculations presented by Hopp & Spearman (2008). In the testing process, the model generated slightly different results from the ones presented in the book. The explanation of this could be the fact that the modeling parameters of the constructed model were slightly different from those presented in the book. In particular, the constructed model took into account lead-time variability, while the case from the Hopp & Spearman (2008) assumed the usage of a constant lead time. Nevertheless, the variability in the results was not significant and taking into account the difference of the modeling parameters, it could be ignored.

In addition, the final validation of the model was done under consideration of the BB Inc. management, when the model was tested with the actual input data. During the testing with the actual input parameters, the model was able to produce appropriate results and, therefore, it was validated by the management of BB Inc. and the author.

5. Discussion and Analysis of results

The purpose of the final chapter is to analyze the conducted work and reveal important aspects concerning the aim of the research, the constructed model, analysis of the research results and possibilities for potential improvement.

5.1 Analysis of constructed model

5.1.1 Conceptual vs. Executable version

The initial idea of the model's development was so that the executable version was constructed based on the parameters and functions that the author specified in the conceptual stage. Indeed, after the development phase, the executable model included all the parameters specified before, and moreover, it was capable of functioning in the way, predefined by the author. All in all, the final version of the model included five Excel spreadsheets with three calculators and two database lists inside. The functional parameters of the model, such as transition and removal of the data between spreadsheets, was developed with the VBA code capabilities, as well as the mathematical mechanism of the calculators, which computed the decision variables. In total, the model contained almost 60 mathematical formulas and 100 lines of the VBA code. Another feature that was developed in the model was the protection of the cells. The reason for that is the complexity that relates to the mathematical mechanism. In order to avoid the removal of the formulas due to the accidental 'clicks,' the author decided to hide them. This feature increases the user friendliness of the model and allows to avoid critical mistakes, which could lead to the malfunction of the

model. As a result, the functionality, mathematical mechanism, user interface design and user-algorithm of the executable version completely corresponded to the conceptual version.

5.1.2 Robustness of the results

The robustness of the model's results completely depends on the modeling parameters, input data used in the calculations and mathematical mechanism. During the validation stage, the mathematical mechanism proved to be robust and produced adequate results with the dummy values from the case-calculation obtained from Hopp & Spearman (2008) and also with the input parameters provided by the Black Bruin Inc. During the testing stage, the behavior of the model was acceptable. Therefore, it could be considered a reliable solution for the inventory control calculation at least for the beginning period of operations with the On-Demand control system. However, the robustness of the results obtained from the model in long-term perspective is questionable due to the essence of some modeling parameters and input data. In the given case, the most important elements of the input data are values of the costs and pattern of the demand and lead time.

Demand modelling.

The pattern of demand has a crucial significance for the computation of optimal order quantity and reorder point. In the current research, the forecasting of demand was not the question. In particular, the value was based on complete estimation. However, in order to improve the accuracy and reliability of results, it is advisable to implement more sophisticated forecasting techniques. Otherwise, at the current stage of research, the pattern of demand could be one of the main reasons for the inaccuracy of the results.

Modelling lead time variability.

Lead time variability is another important modeling element, which has a significant effect on the computation of the reorder point. In the given model, the standard deviation of lead time is computed as the square root of the mean lead-time value. Therefore, if the mean value of the lead-time corresponds to 63 days, then the standard deviation of the lead time is approximately eight days. In the reality of operations, this statement could not always be accurate. In common practice, suppliers often experience quite severe delays, which may extend over more than the period of one week. Thus, this aspect has to be checked against real operations and probably developed one more time with another more sophisticated modeling technique.

Modelling of demand variability during lead time.

Analogically with the lead-time variability, the same statement is relevant for the standard deviation of demand during lead time. The standard deviation of demand during lead time is modeled as a square root of the mean demand during lead time. Therefore, in reality, it could be so that the actual deviation of mean demand during lead time could slightly exceed or underrate the modeled value.

However, due to the fact that the On-demand control system development is in the beginning stage and that the management of BB Inc. does not obtain any accurate information, the observations mentioned above are considered to be only assumptions, and they have to be checked against real operations.

Modelling of costs.

The costing part is the most sensitive input data in the constructed model. In spite of the fact that the developed method for the calculation of costs is more advanced than the previous one used in the company, there is still significant room for improvement. At the initial stage of the project, the interest rate factors for non-capital components of inventory holding cost, such as storage space (facility) cost, inventory service (management) cost and inventory risk cost were based on absolute estimations. Therefore, in accordance with the suggested method, the definition of interest rates might require additional attention and development. A similar claim is relevant for the ordering cost. However, the process of calculation for the components of the ordering cost is even more challenging than for the mentioned non-capital related holding costs, since they represent processes, which in practice are really difficult to evaluate precisely from the financial point of view. At this point, the ordering cost is considered to be one of the most challenging parts the inventory costing model.

The capital cost, which is actually the largest element of the inventory holding cost, could be described as the most robust and reliable costing element in the constructed model. The value of the primary factor that influences the magnitude of the capital cost (WACC - weighted average cost of capital) was obtained from the studies conducted by a financial consulting organization, WACC Expert. The organization has collected statistical data from all over the world and from different industry areas. Furthermore, it reviews new statistics annually. According to WACC Expert, the value of WACC in the sector of technology in Finland varies from **8.91%** to **12.32%**. Thus, it was decided to utilize the average value of WACC, which corresponds to **10.24%**.

The last, but also crucially important, elements of the total cost are backorder and stock out. The estimation of them is also a rather challenging task. However, it could be done more precisely with the help of a sensitivity

analysis. The detailed description of the sensitivity analysis for the backorder and stock out cost is provided in Chapters 4.5.3 and 4.5.5.

In summary, despite the fact that the suggested calculation method certainly has a number of drawbacks, it definitely has improved the inventory cost analysis techniques of BB Inc. and could be reliably implemented in inventory control at least at the initial stage of operations. Nevertheless, the robustness of the results and the implementation of constructed inventory model in long term should be considered critically, and emphasis should be placed on the development of the most uncertain elements.

5.1.3 Advantages and Disadvantages of the model

Finally, the following is a summary of the capabilities and drawbacks of the constructed model.

Positive features that the model provides:

- 1) The model is capable of computing the values of the optimal reorder point and the optimal order quantity.
- 2) The model includes two approaches for calculation: the backorder cost approach and stock out cost.
- 3) The model computes an optimized total cost of the inventory.
- 4) The model shows the values of performance measures and inventory components: safety stock, base stock level, ordering frequency, service level, average backorder/stock out level, average inventory level and the values of the costing components.
- 5) The model allows the inventory manager to make adjustments of order quantity and the reorder point, and it shows the effect of the change on the total cost value and performance measures.
- 6) The model includes a sensitivity analysis of the backorder and stock out cost based on color coding and shows whether their values are estimated as too big or too low.

Flaws of the model:

- 1) The degree of uncertainty in the costing part is significant and therefore it could affect the accuracy of the results.
- 2) The behavior of certain modelling assumptions could differ from the behavior of the actual operations in practice and therefore could influence the accuracy of the decision variables.

5.1.4 Sensitivity of the model

Sensitivity analysis of the model has been conducted and revealed the most sensitive modelling elements. In particular, the sensitivity of the most uncertain elements has been tested and the effect of their modification on the decision variables and the performance measures has been identified. The most challenging in estimation and the most uncertain elements turned out to be the most sensitive elements as well. Particularly, the cost of backorder and the cost of stockout have crucial impact on the optimal order quantity and the optimal reorder point as well as on the service level value. In addition, the other costing elements have also turned out to be quite sensitive. Nevertheless, the magnitudes of these elements are assumed to be more or less precisely estimated, therefore the requirement to indicate their degree of uncertainty is not necessary.

The sensitivity analysis in the given model is not capable to provide the precise values of backorder and stock out costs. However, with the help of sensitivity analysis the model is capable to indicate the numerical range of the costs when they produce relatively adequate values of the decision variables and the performance measures. The mechanism of sensitivity analysis is thoroughly explained in the Chapter 4.5.5.

5.2 Discussion of research questions

The general focus of this thesis included both aspects of research: theoretical and practical. The theoretical objective of the research was to study, understand, analyze and compare various approaches to materials management and inventory control. While the practical objective of the thesis was to apply the gained knowledge to the company case and develop a tool regarding decision-making in the inventory control procedures. The overall aim of the research was to provide answers to three research questions. The main research question was:

1) *What inventory control system could best deal with the uncertainty of the demand and fit into the operations of Black Bruin Inc?*

To answer this question, the author studied the scientific literature on materials management and inventory approaches starting from the basic concepts and gradually constricting it to the focus of inventory control techniques. The theoretical definitions of the basic inventory concepts, as well as materials management, brought significant understanding of the development of an inventory system and provided a strong clue of the benefits that could be achieved with it. Specifically, the author discovered that a proper implementation of materials management approaches is significantly important for the operational efficiency and financial performance of the enterprise. The concept of materials management is rather a broad adumbration, which includes many functions, and inventory control is one of them. The inventory is a crucially important asset of the organization. Properly implemented inventory control could vastly improve the operational performance of the enterprise, and on the other hand, incorrect application of inventory techniques could lead to significant loss of capital and decrease operational performance while in some cases even to the loss of customers. The decision regarding an appropriate inventory

solution is rather sophisticated. In particular, it must take into account not only the operational features as the pattern of lead time and demand, but it also has to consider a number of strategical and tactical aspects, such as the business model of an enterprise or the strategical role of an inventory for the company. In order to achieve high efficiency concerning inventory operations, all of the aspects must be taken into account and aligned according to the strategy of the organization and specificity of operations. Over the years, many methodologies for inventory control have been developed. In the actual research, the author studied, analyzed and compared traditional inventory control methods, such as ABC analysis, EOQ (Economical Order Quantity model) and more profound inventory solutions. As a consequence, it was revealed that conventional methods, which were developed years ago and considered undeniable concepts, were robust and efficient approaches. However, they were mostly designed for straightforward, primitive cases and often their application to the majority of real problems was not appropriate, since they did not account for many factors of uncertainty in the actual operations. Therefore, the author underlined the necessity to use more sophisticated, profound solutions such as statistical inventory models. In particular, the author studied the Base Stock Level model and the (Q, r) model. The necessity to conduct an analysis of both models was essential since it turned out to be that the mathematical mechanism of BSLM is an integral element of the (Q, r) model, and for the sake of better understanding, it was necessary to define the BSLM as well. Consequently, based on the information obtained from the literature review and taking into account the operational features of BB Inc., the researcher identified that the (Q, r) model was the most appropriate solution for the BB Inc. case. Eventually, the author constructed the inventory control tool in Excel spreadsheets, where the general mechanics of the (Q, r) model was replicated. The relevance of the constructed inventory solution tool was tested and accepted by the management of BB Inc.

Indeed, the constructed model was able to produce adequate results, and on the final stage of the research, it was tested with the actual input parameters. During the testing, the model generated adequate results of the optimal order quantity and reorder point with respect to the service level and the total cost of inventory. In addition, the author tested the model during the validation stage with the values taken from the proposed calculation cases in Hopp & Spearman (2008). At this point, the model also produced adequate outputs. However, the utilization of the model in real processes could differ from the cases proposed in the literature. Therefore, there is still plenty of room for improvement concerning certain modeling parameters and aspects. The analysis of the model has been thoroughly described in the chapter 5.1 and the most sensitive parts identified. The utilization of the model in a long-term perspective is questionable at that point. Whether the enterprise is willing to implement the solution in the long-term perspective, they should analyze its behavior against the actual inventory operations and identify and develop the most sensitive modeling parameters. Nevertheless, the constructed model could be seen as a robust solution for inventory control at least for the starting period.

Besides the identification of an appropriate inventory solution for the given case, the secondary aim was to define and calculate the costs of inventory. Therefore, the second research question implied the following:

2) *What would be the costs of the inventory?*

During the research process, the author discovered that cost analysis was a rather complicated procedure, and for the sake of its successful implementation, it was necessary to conduct another study in the field of management accounting. Namely, the author discovered that the actual inventory costs were going far beyond the conventional definitions, such as inventory holding cost or ordering cost. Particularly, the traditional definitions of costs related to inventory (holding cost, ordering cost, stock

out cost, etc.) actually consist of several components, where each component represents the expenses related to the performance of a certain process or activity required to maintain the life cycle of the inventory. A number of complicated issues arise from this statement. One of them is the mathematical definition of the costing components. Since the key drivers beyond them are represented by intangible measures, such as the cost of the clerical process related to ordering of an item or the cost of the inbound logistics process, their precise calculation requires implementation of advanced knowledge of management accounting or other sophisticated financial techniques. A great deal of research has been conducted in the area of inventory accounting, and many solutions have been proposed. The author studied several management accounting resources, for example "Handbook of Management Accounting Research: Volume 1" by Cristopher S. Chapman, Anthony G. Hopwood, Michael D. Shields and discovered a number of potential options. However, at this point, another challenge occurred that was related to Black Bruin's, an SME's, ability to put these sophisticated solutions into practice. In fact, it turned out that the implementation of scientific management accounting approaches to a certain SME was almost impossible. The majority of the scientific resources in management accounting does not actually suggest solutions or formulas that might be more or less adequately applied to reality. On the other hand, the suggested options include a great number of different variables and terms that stand far from the peculiarity of operational methods in SMEs. In the particular case of Black Bruin Inc., inventory accounting was implemented in the most simplified way: the financial department only reviewed the value of items stored in the inventory on a monthly basis. Moreover, such terms as the cost of equity, corporate tax rate, cost of debt etc. were completely unfamiliar to the financial department of Black Bruin Inc. This issue was one of the key findings for the author, as he discovered the problem of compatibility between scientific methods and real working

cases. The essence of the issue is really relevant and perturbing since the cost analysis of such assets as inventory is an important part of profitability improvement and operational development for organizations. In case the enterprise does not take into account such significant factors as calculation of inventory costs, the identification of their components and key drivers, they simply could lose a significant amount of money related to expenses that they are not even aware of. At this point, the cost analysis was the crucial factor for this research, as the aim of the inventory model was to improve the operational efficiency of BB Inc., which was literally impossible without taking the costs into account. Accordingly, with the approval of BB Inc.'s management, the author implemented another approach for the calculation of inventory costs. Generally, the detailed approach to cost calculation is thoroughly described in Chapters 3.2.8 and 4.5.3. Eventually, the author was able to establish the calculation method for the inventory costs, but it was necessary to take into account the factor of uncertainty. In the given case, the suggested calculation method definitely improved the cost analysis of BB Inc.'s inventory asset, but nonetheless, the obtained numbers could not be treated as absolutely veracious values. On the contrary, they represented approximate numbers, which the values of the inventory costs could take. Particularly, this issue was related to the interest rates of the costing components and where they were coming from. In the case of BB Inc., it was decided that the responsibility for establishing interest rates fell on the side of BB Inc. management, and, at this point, the important aspect was how they would implement the calculation. Nevertheless, even if the method could not be accounted for as an absolutely accurate one, the inventory model proved that it was able to generate adequate outputs with the suggested calculation method.

The third research question was:

3) *If it is possible, how the costs could be optimally reduced concerning the constraints?*

In the given research, the constructed inventory model included the part of costs optimization. However, the optimization process was not possible to implement through the linear or nonlinear optimization method using capabilities of Solver Add-in in excel software. On the other hand, the total cost function of inventory was optimized through its' differentiation with respect to the optimal order quantity, the optimal reorder point and setting the result equal to zero yields. Therefore, the variables that produce the minimum objective value of the total cost function are referred to the values of Q and r. The constraint in this situation is the service level the company is willing to reach. In this particular case, the desired service level was specified by the management of BB Inc. and its' value corresponded to the ***%, which in many cases does not imply the lowest possible total cost with respect to Q and r. Accordingly, the desired service level as a constraint would not always allow the organization to attain the minimum value of the total cost.

However, depending on the values of input parameters, the model computes service level, which is 2%-15% lower than the desired value. In appliance with the constructed model, there are two ways, how the desired value of service level could be reached in this situation, either by the increase in the value Q or the value of r. At this point, the researcher has made an important observation that increase in the value of Q or r influences the total cost function differently. The increase in the value of Q produces smaller rise in the total cost function, but at the same time it reflects to the smaller increase in the value of service level, while the increase in value of r generates the larger increase in the value of the total cost function and in the value of service level as well. At this perspective, the optimization procedure could be considered as the tradeoff between the optimal increase in either value Q or value of r, which generates the smaller value of the total cost function and at the same time reaches the desired service level.

However, this procedure has to be done straight at a site, for each item separately, since each item always obtains different input parameter and therefore it results in an individual calculation output. All in all, the aim of this thesis was to create a tool that includes particular features and supports BB Inc. in the decision-making process. Consequently, the responsibility of the author was to provide a tool, while the responsibility of the management was to make the decision.

5.3 Advices for Management

During the validation stage, the author has made several observations, which might be useful for the model's users. For example, backorder approach calculator turned out to be more accurate in appliance with the case sample calculation from Hopp & Spearman (2008) than stockout calculator. In particular, stockout calculator tends slightly overrate the measure of service level. The explanation of this is the fact that integral calculation in excel is limited, therefore the author had to use the approximation type 2 from Hopp & Spearman (2008), which might slightly overestimate the actual value. The stock out calculator could also be used for planning purposes, however, in this case, the author advice to subtract **0.3-0.4%** from the computed service level measure.

The next observation is related to the planning horizon of the model. Initially, the standard planning horizon was considered to be one year. However, during the validation stage, the author has made an important notice that model is also capable of generating adequate outputs for shorter planning periods. For example, in case if the demand is unevenly distributed over the year, the planning horizon could be shortened to half a year period, and decision variables could be computed based on the demand for this period.

The last observation of the author relates to the dependency of safety stock and service level from the magnitude of backorder and stock out cost. If the magnitude of either backorder or stockout cost is estimated to be too low, it generates safety stock with negative sign and too low service level value (less than 90%). Generally, the negative sign of safety stock means that value of mean demand is higher than the value of the optimal reorder point. In this situation, there are two options: either increase the value of backorder/stockout cost and see actual expenses related to poor customer service as well as new values of service level and safety stock or if the manager is willing to continue with the chosen magnitude of backorder/stockout cost, but he wants to increase the value of service level, he could do it through increase of Q or r . At this point, important notice is that increase of Q does not influence the magnitude of the safety stock, but increases the service level, while the increase of r with one unit increases the magnitude of safety stock with one unit respectively and also generates the increase in service level. In accordance with this statement, the manager could increase the service level to required value through the increase of Q , but the safety stock will still obtain the negative sign. The author advice to avoid cases when the safety stock is negative since even if the value of service level would be adjusted to permissible level, the probability of not responding the customer order is still high due to the low reorder point. On the other hand, the suggestion of the author is to increase the value of the reorder point so that the magnitude of safety stock would be at least one unit and then, if the service level value is still lower than required, it could be increased through the increase in the magnitude of Q . All in all, the idea is to avoid the safety stock with negative sign, even if the service level high.

5.4 Suggestions for further research and improvements

The research and model in the scope of this thesis could also be used for further research opportunities and certain improvements. First of all, the constructed model includes only one option of modeling assumptions. For instance, the service level is modeled according to one particular type of distribution. In prospect, it would be interesting to include in the model several options for service level modeling and other modeling assumptions so that the user could choose an appropriate type of probability distribution according to the case and the type of demand. In addition, another modeling features could also be modeled with several options according to the operational characteristics of the case.

Secondly, the constructed model provides the option of decision variables calculation and also visualize the relationship between the magnitude of the costs and decision variables. However, even if the user is able to compute the optimal values of Q and r , the behavior of inventory under these conditions is still a question. According to this statement, a significant value could be added to the model if the option of inventory simulation with calculated the optimal decision variables would be included in the model and user could see the actual behavior of inventory according to certain scenarios. Graphical visualization of simulation process could also bring significant understanding concerning chosen inventory system.

Furthermore, the given inventory model provides the user with an option to see the database with cumulated inventory and costing data that was computed through the model. However, the capabilities of the database with the calculated data is quite limited in the given case, and it only shows the statistical information of inventory components, costs and performance measures. It would be interesting to include in the model inventory-tracking

option so that the user could see not only the statistical data of inventory according to the calculation but also the real data recorded from the actual inventory operations. This option could significantly improve decision-making process regarding inventory operations since it will provide the user with the possibility to see the actual inventory behavior and based on that improve the mechanics of constructed inventory model. In addition, it will provide the possibility to compute such performance measures as inventory turnover ratio, inventory throughput time, etc. that will definitely be useful for better control of inventory operations.

Finally, another suggestion that could be added to the model is an option of automatic data transition from the excel spreadsheets to the ERP system. However, this opportunity for improvement relates more to the research in the field of information technology, rather than materials management and inventory control. Nevertheless, this option could also improve the inventory-data transition processes and reduce the time required for manual upload of the data to ERP system.

6. Conclusion

To sum up the research, it is necessary to tell about the importance of mathematical modeling application to the operational problems when the decision has to be made under uncertain conditions, for instance, inventory control. At the beginning of the industrial era, first computational solutions such as ABC analysis have been proposed. However, from these days on, the majority of enterprises were mainly making decisions concerning operational problems based on the communication between departments and members in supply chains. However, this approach has a huge number of drawbacks which result in an inefficient operational performance and irrational use of resources. Many companies consider mathematical modeling of operations

as a very complicated and an unnecessary tool. However, on the example of this thesis work it could be clearly seen how the application of mathematics to the inventory operations of SME such as Black Bruin Inc. could significantly simplify and improve the decision-making process and also increase the degree of reliability. Indeed, the solution developed during this research work has created significant value for the company and the result of the research could be fairly considered as a positive outcome.

Nevertheless, it still contains a lot of points for improvement, which should be considered for further research opportunities.

The potential future improvement of the solution might greatly help the company to perform even more efficient operations and as a result to assure successful future and become up-to-date, technologically-developed enterprise, which is capable to operate and provide high-quality products in the modern industrial business.

Acknowledgements

Finally, I would like to add that I sincerely appreciate the support from all people that made this thesis opportunity possible for me. Firstly, I would like to give thanks to my main supervisor from JAMK Mr. Juha Sipilä for assistance and constant guidance in the management of the thesis work. Secondly, I would like to thank my second supervisor Mr. Pasi Lehtola for necessary criticism and an indispensable assistance in mathematics. Thirdly, I would like to thank all the logistics engineering department of JAMK and especially Mr. Hannu Lähdevaara, for the necessary consultancy and advice. Finally, I would like to give thanks to Mr. Seppo Koiranen, Mr. Jakke Palonen, all On-Demand control system project team and the rest of people working at Black Bruin Inc. for giving me an opportunity to complete this thesis work.

Lastly, I would like to give thanks to the friends and people who supported

me in difficult moments encouraged to move forward and overcome difficulties on my way.

All of you have provided me with an irreplaceable assistance that I faithfully appreciate.

Great thanks to all of you,

Anton Saukkonen.

References.

Advantages and Disadvantages of Qualitative Research N.d. OCCUPYTHEORY 21 April 2014. Accessed on 14.1.2017. Retrieved from <http://occupytheory.org/advantages-and-disadvantages-of-qualitative-research/>

Arnold, J.R.T., Stephen, N., Chapman & Lloyd M. Clive. (2007). Introduction to Materials Management. 6th p. New Jersey: Pearson Prentice Hall.

Bharath, R. Prakash. G.S. (2014) Lead time Reduction Using Lean Manufacturing Principles for Delivery Valve Production. Global Journal of Finance and Management. ISSN 0975-6477 Volume 6, Number 1 (2014), 35-40. Assessed on 15 February 2017. Retrieved from http://www.ripublication.com/gjfm-spl/gjfmv6n1_06.pdf

Braun, V., Clarke, V. (2013). Successful Qualitative Research: A practical guide for beginners. London: SAGE Publications Ltd.

Caplice, C. (2006). Inventory Management Multi-Items and Multi-Echelon. Lecture in Inventory. Massachusetts Institute of Technology, Logistic Systems, ESD.260/15.770/1.260. Assessed on 10 February 2017. Retrieved from <https://ocw.mit.edu/courses/engineering-systems-division/esd-260j-logistics-systems-fall-2006/lecture-notes/lect17.pdf>

Carmichael, D.G. (2013). Problem Solving for Engineers. CRC Press. Taylor & Francis Group, LLC. 375-380.

Creswell, J. (2014). Qualitative, Quantitative and Mixed Methods Approaches, Research Design. London: SAGE Publications Ltd.

Dahlberg, L., McCaig, C. (2010). Practical Research and Evaluation. London: SAGE Publications Ltd.

Dijk, E.v., Leeuw, S.d. & Durlinger, P. (2007). Voorraadbeheer in perspectief:

Zeven invalshoeken van het vak (*Inventory in perspective: Seven perspectives of the trade*). Deventer: Slimstock B.V.

DME for peace: Section 1.3 *Quantitative Research Module*. N.d. Search for Common Ground. Accessed on 14.1.2017. Retrieved from <http://dmeforpeace.org/sites/default/files/1.3%20Quantitative%20Research.pdf>

Fernandes, N. (2014). *Finance for Executives: A Practical Guide for Managers*. 32.

Frechtling Westat, J. 2002. *The User-Friendly Handbook for Project Evaluation*. The National Science Foundation Directorate for Education & Human Resources Division of Research, Evaluation, and Communication, January 2002. Accessed on 19.1.2017. Retrieved from <https://www.nsf.gov/pubs/2002/nsf02057/nsf02057.pdf>

Gilland, M. (2011) Why forecasts are wrong: Unforecastable demand. SAS Blogs. Assessed on 23 February 2017. Retrieved from <http://blogs.sas.com/content/forecasting/2011/11/08/why-forecasts-are-wrong-unforecastable-demand/>

Goetsch, D.L. & Davis, S.B. (2010). *Quality Management for Organizational Excellence: Introduction to Total Quality*, 6th Edition. 538-579. Pearson, ISBN: 978-0-13-501967-2.

Gwynne, R. (2011). *Warehouse Management: A Complete Guide to Improve Efficiency and Minimizing Costs*. Kogan Page Publishers. 62-64.

Harris, F.W. (1913). How Many Parts to Make at Once. *Factory, The Magazine of Management* vol. 10, no. 2, 135–136, 1913.

Hillier, F.S., Lieberman, G.J. (2010). *Introduction to Operations Research* 9th Edition. McGraw-Hill Higher Education, 2010.

Hoekstra, S. & Romme, J.H., (1991). *Integral logistic structures: developing customer-oriented good flow*. London: McGraw-Hill.

Hopp, W.J, Spearman, M.L. (2008) *Factory Physics* 3rd Edition. ISBN 978-1-57766-739-1. 49-92.

Hoppe, M. (2008). *Inventory Optimization with SAP*. Galileo Press. 373.

Inventory Definition N.d. INVESTOPEDIA. Accessed on 23.2.2017. Retrieved from <http://www.investopedia.com/terms/i/inventory.asp>

Jessop, D., Morrison, A. (1994). *Storage and Supply of Materials* 6th ed. London: Pitman Publishing 1994.

Johnston. K. N.d. *The Advantages & Disadvantages of Economical Order Quantity (EOQ)*. Assessed on 8 March 2017. Retrieved from

<http://yourbusiness.azcentral.com/advantages-disadvantages-economic-order-quantity-eoq-28712.html>

Jonsson, P. (2008). Logistics and Supply Chain Management. Maidenhead: McGraw-Hill Higher Education.

Kärkkäinen, M. (2015) Top Challenges in demand forecasting. Relex. Supply Chain Thinking Blog. Demand Forecasting. Assessed On 23 February. Retrieved from <https://www.relexsolutions.com/top-challenges-in-demand-forecasting/>

Kiisler, A. (2014) Inventory Management – Basic Concepts. Lecture Materials. L-Consult OÜ, LogOn Train Summer School, 30.06.-04.07.2014 Accessed on 14 February 2017. Retrieved from <http://www.vkok.ee/logontrain/wp-content/uploads/2014/03/Riga-3-july-2014.pdf>

King, L. P. (2011). Crack the code. Understanding safety stock and mastering its equations. APICS magazine 1 July-August 2011. Assessed on 19 February 2017. Retrieved from http://web.mit.edu/2.810/www/files/readings/King_SafetyStock.pdf

Koiranen, S. (2017). Technical Director. Black Bruin Inc. Interviews 2017.

Koivula, L. (2015). Modelling Supply Chain Costs in Automotive Industry. The case of Valmet Automotive. Master Thesis. Aalto University, School of Business, Department of Information and Service Economy. Assessed on 31 January 2017.

Lähdeväärä, H. (2014). JAMK University of Applied Sciences, School of Technology, Degree Program in Logistics Engineering, Lecture in Supply Network Management in Autumn 2016.

Lawson, D. Marion, G. (2008). An Introduction to Mathematical Modelling. Bioinformatics and Statistics Scotland. 1-3. Assessed on 14 April 2017. Retrieved from https://people.maths.bris.ac.uk/~madjl/course_text.pdf

Lehtola, P. (2017). Principal Lecturer. JAMK University of Applied Sciences, School of Technology, Degree Program in Logistics Engineering. Interviews 2017.

Magad, E.L. (1995). Total Materials Management: Achieving Maximum Profits Through Materials/Logistics Operations. 2nd Edition. Secaucus, New Jersey, U.S.A.: Kluwer Academic. xi-xii. ISBN 10: 0412065010/ISBN 13: 9780412065019

Materials Management Definition N.d. BusinessDictionary. Accessed on 21.2.2017. Retrieved from <http://www.businessdictionary.com/definition/materials-management.html>

- Mentzer, J.T., Moon, M.A., Estampe, D., Margolis G. (2006) Understanding Global Supply Chains: Demand Management. Assessed on 28 February 2017. Retrieved from https://www.corwin.com/sites/default/files/upm-binaries/11204_Chapter_5.pdf
- Mishra, R. (2007). Materials Management. New Delphi: Excel Books. 4-7, 16-18
- MSG Management Study Guide. N.d. Types of Inventories – Independent and Dependent Demand Inventories. Assessed on 28 February 2017. Retrieved from <http://www.managementstudyguide.com/types-of-inventories.htm>
- Muckstadt, J.A., Sapro, A. (2010). Principles of Inventory Management: When you are down to four, Order more. Springer Series in Operations Research and Financial Engineering.
- Palonen, J. (2017). Laboratory Manager. Black Bruin Inc. Interviews 2017.
- Piippaa, R. (2017). Supply Chain Manager. Black Bruin Inc. Interviews 2017.
- Radasanu, A.C. (2016). Inventory Management, Service Level and Safety Stock. Alexandru Ioan Cuza University, Journal of Public Administration, Finance and Law 149-150. Assessed on 29 April 2017. Retrieved from [http://www.jopafl.com/uploads/issue9/INVENTORY MANAGEMENT SERVICE LEVEL AND SAFETY STOCK.pdf](http://www.jopafl.com/uploads/issue9/INVENTORY_MANAGEMENT_SERVICE_LEVEL_AND_SAFETY_STOCK.pdf)
- Rardin, R.L. (2017). Optimization in Operations Research 2nd Edition. Pearson Higher Education 1998, Inc. Hoboken, NJ 07030. 3-4.
- Ravinder, H. Misra, R.B. (2014) ABC Analysis for Inventory Management: Bridging The Gap Between Research And Classroom. American Journal of Business Education. Third Quarter 2014, Volume 7, Number 3. Assessed on 28 February 2017. Retrieved from <http://files.eric.ed.gov/fulltext/EJ1053624.pdf>
- Sheldon, D.H. (2008). Lean Materials Planning and Execution: A guide to Internal and External Supply Management Excellence. J Ross Publishing 2008. 35-39.
- Silver, E.A., Pyke, D.F., Peterson, R. (1998). Inventory management and production planning and scheduling. New York: John Wiley & Sons 1998.
- Sipilä, J. (2017). Senior Lecturer. JAMK University of Applied Sciences, School of Technology, Degree Program in Logistics Engineering. Interviews 2017.
- Sjoman, J. (2016). Materiaalilogistiikan mallinnus ja optimointi. Case LNGTainer Oy. (*Materials logistics Modelling and Optimization*). Bachelor Thesis. JAMK University of Applied Sciences, School of Technology, Degree program in Logistics Engineering. Assessed on 16 January 2017. Retrieved

from

http://www.theseus.fi/bitstream/handle/10024/115669/Sjoman_Juhani.pdf?sequence=1

Snyman, J.A. (2005). Practical Mathematical Optimization: An Introduction to Basic Optimization Theory and Classic and New Gradient-Based Algorithms. Springer Science + Business Media, Inc. 19. Accessed on 19 April 2017.

Retrieved from http://cindy.informatik.uni-bremen.de/cosy/teaching/CM_2011/fitting/snyman_05.pdf

Spearman, M. L. (2015). Beyond ABC. Factory Physics Inc.

Tersine, R.J. (1994) Principles of inventory and materials management. PTR Prentice-Hall Inc., Upper Saddle River. 591.

Timme, S.G., Timme, C.W. (2003) The Real Cost of Holding Inventory. Supply Chain Management Review, 7/1/2003. Assessed on 11 March 2017.

Retrieved from <http://www.readbag.com/mcasolutions-pdf-cost-of-inventory>

Treacy, M. Wiersema, F., (1993). Customer intimacy and other value disciplines. Harvard Business Review, 71(1), 84-93.

van Heck, G. (2009) Inventory Management: Introducing a Framework to assess Operational Performance. Master Thesis. Delft University of Technology, System Engineering, Policy Analysis and Management. 42-44. Assessed on 1 March 2017.

Vermorel, E. (2013). Inventory costs: Definition and Formula. Lokad: Better Supply Chain Solutions. Quantitative Supply Chain Optimization Software. Knowledge Base. Assessed on 10 March 2017. Retrieved from

<https://www.lokad.com/definition-inventory-costs>

Vollmann, T.E., William, L. B. & Whybark, D.C. (1997). Manufacturing Planning and Control Systems. 4th Edition. Boston: McGraw-Hill Publ.Comp.

Vrat, P. (2014). Materials Management. Springer Texts in Business and Economics, 21-23, 29-33. DOI 10.1007/978-81-322-1970-5_2

Walliman, N. (2011). Research Methods the basics. Routledge 2011.

Accessed on 14.1.2017. Retrieved from

http://uni.delf.pro/uploads/7/1/0/7/7107980/research_methods_the_basics.pdf

Waters, D. (2003) Inventory Control and Management: 2nd edition. John Wiley & Sons Ltd.

Wild, T., (2002). Best Practice in Inventory Management 2nd edition. Oxford: Butterworth-Heinemann (imprint of Elsevier).

Willoughby, K.A. N.d. The Disposal of Excess Stock. University of Saskatchewan, Edwards School of Business, Department of Finance and Management Science. Assessed on 3 March 2017. Retrieved from <https://www.edwards.usask.ca/faculty/Keith%20Willoughby/files/Excess%20stock%20disposal.pdf>

Zipkin, P. (2000) Foundations of Inventory Management. McGraw-Hill/Irwin 2000-01-24, 2000.

Appendices.

Appendix 1. Inventory Model developed in Excel with VBA (**Not available for public access**)

Appendix 2. Manual for the model Version 1.2 (**Not available for public access**)

Appendix 3. Excel File with WACC calculation in Technology sector of Finland